

Prof. H. A. Dole
Three Dollars per Annum, in Advance. Single Copies, Twenty-Five Cents.

VOL. III.

No. III.

THE

MATHEMATICAL MONTHLY.

DECEMBER, 1860.

EDITED BY

J. D. RUNKLE, A.M., A.A.S.

CAMBRIDGE:

PUBLISHED BY SEVER AND FRANCIS.

NEW YORK:

A. S. BARNES AND BURR.

LONDON: TRÜBNER & CO.

1860.

Editorial Items.

Prizes for Vol. III. offered to all Students.

FOR SOLUTIONS. — A 1st Prize, \$7.50; 2d, \$5; 3d, \$3; 4th, Vol. III. The third and fourth prizes are confined to beginners for solutions of Problems I. and II.; but as the best solutions only will be published, advanced students are desired to send solutions of all the problems.

FOR ESSAYS. — A 1st Prize, \$50; 2d, \$40; 3d, \$30; 4th, \$20; 5th, \$10. The first and second of these prizes are open to all competitors. Instead of cash prizes, we propose to make them all payable in mathematical books. This change will make the prizes less burdensome to us while it will be appropriate, and we believe quite as acceptable to competitors, to receive standard mathematical books, — such as BOWDITCH's edition of *Mécanique Céleste*, PEIRCE's *Analytic Mechanics*, BARTLETT's *Analytic Mechanics*, DAVIS's edition of *Theoria Motus*, DAVIES and PECK's *Mathematical Dictionary*, CHAUVENET's *Analytic Trigonometry*. But selections may be made from SEVER & FRANCIS's list, and desired works not found there will be ordered. To those preferring standard works in other departments, we offer WORCESTER's *Quarto Dictionary* and the fine editions of HUME, GIBBON, LAMB, and MACAULAY, HAMILTON's *Metaphysics and Logic*, and APPLETON's *New American Encyclopædia*, described in our advertising pages. To this list we may hereafter add editions of other standard authors, if desired.

A Generous Offer.

It gives us great pleasure to announce that "A Friend of the Mathematical Monthly Prizes," whose name we are not permitted to give, wishes to assume the responsibility of the Prizes for the solutions of the Problems in the number for December. He says, "The Monthly has been of great value to me. It has been my tutor, and certainly the tuition is cheap. I ought to be able now to contribute something more to its sustenance." The solutions will be due February 1, 1861, and the Report will be published in the March number. We hope to receive a large number of solutions as evidence of an exciting contest for these Prizes. It will be seen (Editorial Items), that the times for the solutions of the September and October Problems are extended one month. Hereafter we shall issue the Monthly promptly on or before the first of the month.

Portrait of Sir John Herschel.

Through the kindness of MISS MARIA MITCHELL we have received a fine photograph of SIR JOHN HERSCHEL, which we shall have engraved on steel for an early number.

We expect our friends will aid us by increasing our subscription list. See Mathematical Monthly, on seventh page of Advertiser.

Contents.

DECEMBER, 1860.

	PAGE		PAGE
PRIZE PROBLEMS FOR STUDENTS	65	GREATER THAN THE GEOMETRIC — NOTE ON THE	
NOTES AND QUERIES — PERFECT SQUARES — PROBLEM — NOTE ON PRIZE SOLUTION OF PROBLEM III, p. 25 — CORRECTION OF THE METHOD OF COMPUTING SIX PER CENT INTEREST FOR DAYS — SOLUTION OF PROBLEM V., No. IX., VOL. II. — NOTE ON POWERS OF BINOMINALS — THE ARITHMETIC MEAN OF ANY NUMBER OF POSITIVE QUANTITIES IS		SUCCESSIVE DERIVATIVES OF TAN ϕ	66-70
		THE EARTH CONSIDERED AS A SPHEROID OF REVOLUTION — GEODETIC FORMULAS	71
		ON SPHERICAL ANALYSIS	78
		A TREATISE ON DETERMINANTS	86
		MATHEMATICAL MONTHLY NOTICES	90

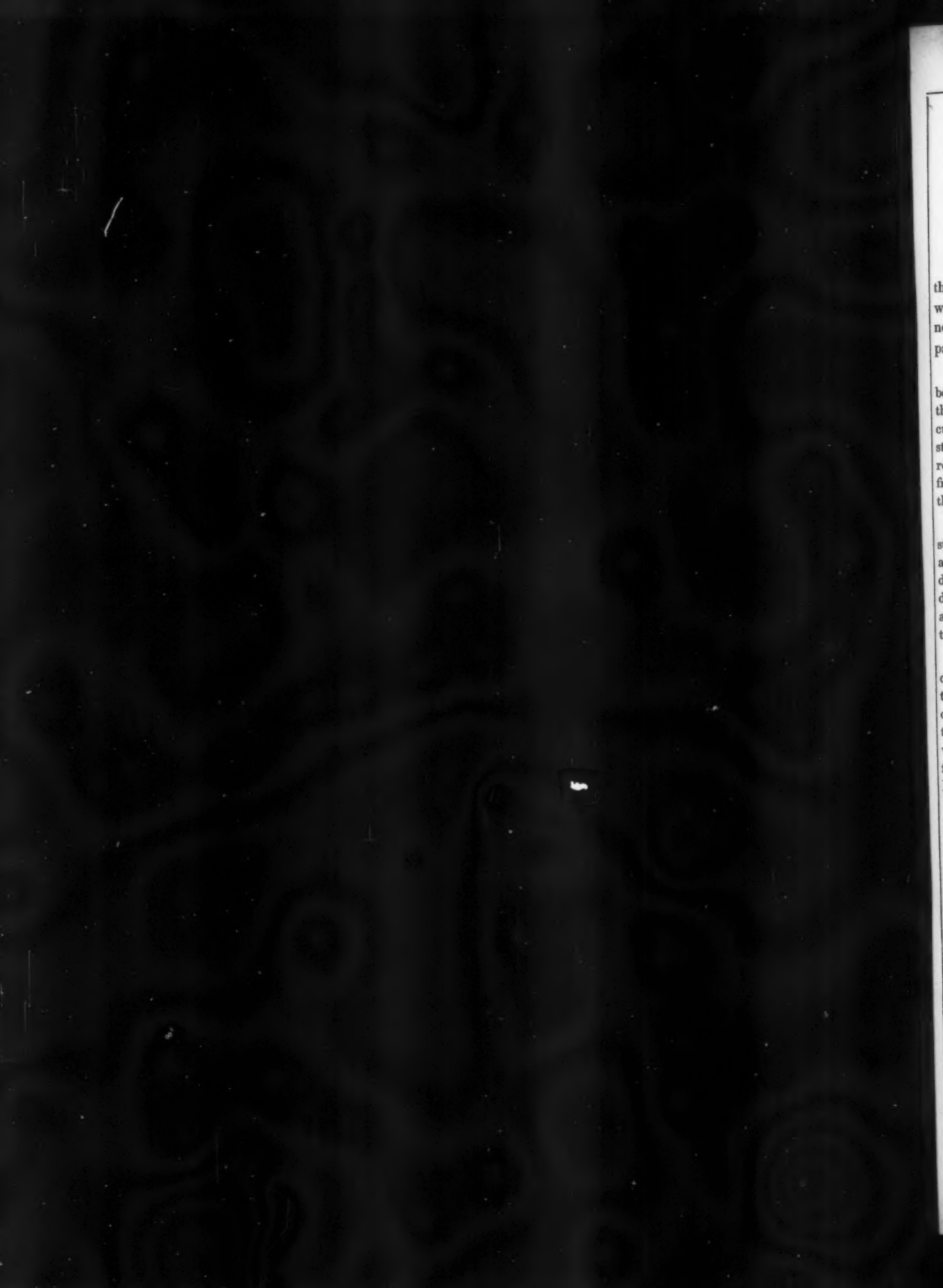
Terms to Subscribers and Advertisers.

No. of Copies to one Address.	Volume.	Terms.	Premium to Clubs.	Advertisement Pages.	First Insertion.	Subsequent Insertions.
1	III.	\$ 3.00		1	\$ 15.00	\$ 12.00
2	III.	5.00		$\frac{1}{2}$	9.00	7.00
3	I., II., III.	6.00		$\frac{1}{4}$	5.00	4.00
5	III.	11.00	III.	$\frac{1}{8}$	3.00	2.00
10	III.	20.00	II., III.			
15	III.	30.00	I., II., III.			

Parties in New York wishing to advertise in the Mathematical Monthly will please send their favors to Messrs. A. S. BARNES AND BURR by the 15th of each month. All club subscriptions and business communications must be addressed to

SEVER AND FRANCIS, Publishers,
Cambridge, Mass.

University Press, Cambridge: Electrotyped and Printed by Welch, Bigelow, & Co.



th
w
no
po

be
th
cu
st
re
fi
th

s
a
d
a
t

c
t
c
f
v

THE TRIBUNE FOR 1861.

PROSPECTUS.

The XXth Volume of THE WEEKLY TRIBUNE commenced with the issue of September 1. During the past year THE TRIBUNE has been obliged to devote quite a large proportion of its space to Politics, but we shall soon be able to forego political discussion almost entirely, for months if not for years, and devote nearly all our columns to subjects of less intense, but more abiding, interest. Among these, we mean to pay especial attention to

I. EDUCATION.—The whole subject of Education, both Popular and General, will be discussed in our columns throughout the year 1861, and we hope to enlist in that discussion some of the profoundest thinkers and the ablest instructors in our country. It is at once our hope and our resolve that the cause of Education shall receive an impetus from the exertions of THE TRIBUNE in its behalf during the year 1861.

II. AGRICULTURE.—We have been compelled to restrict our elucidations of this great interest throughout 1860, and shall endeavor to atone therefor in 1861. Whatever discovery, deduction, demonstration, is calculated to render the reward of labor devoted to cultivation more ample or more certain, shall receive prompt and full attention.

III. MANUFACTURES, &c.—We hail every invention or enterprise whereby American Capital and Labor are attracted to and advantageously employed in any department of Manufacturing or Mechanical Industry as a real contribution to the Public Weal, insuring ampler, steadier, more convenient, more remunerating markets to the Farmer, with fuller employment and better wages to the Laborer. The Progress of Mining, Iron-making, Steel-making, Cloth-weav-

ing, &c., &c., in our country, and the world, shall be watched and reported by us with an earnest and active sympathy.

IV. FOREIGN AFFAIRS.—We employ the best correspondents in London, Paris, Turin, Berlin, and other European capitals, to transmit us early and accurate advices of the great changes there silently but certainly preparing. In spite of the pressure of Domestic Politics, our News from the Old World is now varied and ample; but we shall have to render it more perfect during the eventful year just before us.

V. HOME NEWS.—We employ regular paid correspondents in California, at the Isthmus of Darien, in the Rocky Mountain Gold Region, and wherever else they seem requisite. From the more accessible portions of our own country, we derive our information mainly from the multifarious correspondents of the Associated Press, from our exchanges, and the occasional letters of intelligent friends. We aim to print the cheapest general newspaper, with the fullest and most authentic summary of useful intelligence, that is anywhere afforded. Hoping to "make each day a critic on the last," and print a better and better paper from year to year, as our means are steadily enlarged through the generous co-operation of our many well-wishers, we solicit and shall labor to deserve a continuance of public favor.

TERMS.

DAILY TRIBUNE, 311 issues per annum,	\$6.00
SEMI-WEEKLY, 104 " " "	\$3.00
WEEKLY, 52 " " "	\$2.00

TO CLUBS.—SEMI-WEEKLY, two copies for \$5, five for \$11.25; ten copies to one address for \$20; and any larger number at the latter rate. For a club of twenty, an extra copy will be sent. For a club of forty, we send THE DAILY TRIBUNE gratis one year.

WEEKLY: Three copies for \$5; five for \$8; ten for \$12, and any larger number at the rate of \$1.20 each per annum, the paper to be addressed to each subscriber. To clubs of twenty we send an extra copy.

Twenty copies to one address for \$20, with one extra to him who sends us the club. For each club of one hundred, THE DAILY TRIBUNE will be sent gratis one year.

Payment always in advance. Address,

THE TRIBUNE, No. 154 Nassau-st., New York.

DODD'S MATHEMATICAL SERIES.

Text-Books for Colleges, Academies, and Schools.

BY PROF. JAMES B. DODD, TRANSYLVANIA UNIVERSITY, KY.

The attention of Instructors is respectfully invited to the books of Prof. Dodd's series, which are works of an established character, and used with satisfaction in prominent institutions of learning throughout the United States and Canada.

	RETAIL PRICE.
ELEMENTARY AND PRACTICAL ARITHMETIC. 12mo. 291 pp. Boards	\$0.50
HIGH-SCHOOL ARITHMETIC. 12mo. 364 pp. Cloth84
KEY TO ARITHMETIC. 12mo. 133 pp. Boards38
ELEMENTS OF ALGEBRA. 12mo. 217 pp. Sheep84
HIGH-SCHOOL ALGEBRA. (Table of Logarithms from 1 to 10,000. 16 pp.) 12mo. 306 pp. Sheep	1.25
KEY TO ALGEBRA. 12mo. 173 pp. Cloth84
ELEMENTS OF GEOMETRY AND MENSURATION. 12mo. 237 pp. Sheep	1.00
TRIGONOMETRY. With applications to SURVEYING, NAVIGATION, etc. 12mo. 212 pp. Tables 80 pp. Sheep	1.25

☞ A Primary Arithmetic, and books to complete the series, are in preparation, and will soon be published.

Dodd's Arithmetics and Geometry are adopted for the public schools of New York, and several other cities of the country.

Descriptions and many excellent commendations in our new Descriptive Catalogue of College and School Books, copies of which will be furnished gratis on application to

PRATT, OAKLEY, & CO., Publishers,

21 Murray Street, New York.

THE NEW AMERICAN CYCLOPÆDIA.

D. APPLETON & CO.,

Nos. 443 & 445 Broadway, New York,

HAVE JUST PUBLISHED

Vol. XI. of the

New American Cyclopædia,

A POPULAR DICTIONARY OF GENERAL KNOWLEDGE,

EDITED BY

GEORGE RIPLEY and CHARLES A. DANA,

Aided by a Numerous Select Corps of Writers from all parts of the United States.

This work is being published in about 16 large octavo volumes, each containing 750 two-column pages.

VOLS. I. TO XI. NOW READY.

To be followed by a new Volume every three months.

Price — In cloth, \$3; sheep, \$3.50; half morocco, \$4; half Russia, \$4.50 each.

From De Bow's Review, New Orleans.

"This great work, published by the Appletons of New York, has reached its eighth volume. It is one of the most valuable Cyclopædians in the world, and for Americans the most valuable. It is con-

tributed to by many of the ablest and best writers of the South, and is as fair to every section of the Union as it is possible for a work to be. It will be completed in about seven other volumes."

The following notice in *Silliman's American Journal of Sciences and Arts*, for March, is from the pen of the venerable senior editor, one of the highest living authorities on the subject:—

"In the American Cyclopædia the articles on Science are numerous and valuable, and elevate the work to the character of a compendium of modern science. These articles are in most cases written with decided ability, and evidently by persons who are familiar with the topics they discuss. While many of the less important subjects are presented with luminous brevity, others are more fully expanded. Among these are many topics of Natural History. Chemistry is presented with its equivalence and laws of combination, illustrated by many of its modern discoveries and practical applications; of the latter, an example is found in the full account of the manufacture of gelatine, of beer, and bricks, and in the ample history of gas-lighting. Neology, voltaic electricity, magnetism, and other departments of pure or applied science, are treated with reasonable fulness. As a literary work the Cyclopædia is written in a pure and chaste style, and exhibits the candor and fairness which should ever adorn a record of universal knowledge. L. S."

OUR SUBSCRIPTION LIST

Already numbers TWELVE THOUSAND names, from every section of the country, which evidences the universal popularity of this NATIONAL WORK.

THE NEW AMERICAN CYCLOPÆDIA

Is sold exclusively by subscription, and Agents have been appointed for almost all sections of the Union. In case, however, Agents are not to be found, we will receive subscriptions and forward copies of the work, free of expense, on receipt of the price. Persons at a distance may send money for one or more volumes at a time, it not being necessary to take the whole eight volumes at once, unless they feel inclined to do so. Those living in the city will find a subscription-book at our store, where their names will be entered, and the volumes, as published, sent to any part of the city.

FOR

THE undersigned will issue a series of volumes under the general title of **Papers for Teachers and Parents**, edited by Henry Barnard, LL. D., devoted to a practical exposition of *Methods of Teaching and School Management* in different countries, with special reference to Primary and Common Schools, and Home Education.

The following volumes are now ready :—

- I. AMERICAN CONTRIBUTIONS TO THE PHILOSOPHY AND PRACTICE OF EDUCATION. By Professor William Russell; Rev. Dr. Hill, President of Antioch College; Rev. Dr. Huntington; Gideon F. Thayer, late Principal of Chauncy Hall School; Rt. Rev. Bishop Burgess, and others. One Volume, 404 pages octavo; bound in cloth, \$1.50; bound in half goat, \$2.00.
- II. OBJECT-TEACHING AND ORAL LESSONS ON SOCIAL SCIENCE AND COMMON THINGS, WITH VARIOUS ILLUSTRATIONS OF THE PRINCIPLES AND PRACTICE OF PRIMARY EDUCATION, AS ADOPTED IN THE MODEL AND TRAINING SCHOOLS OF GREAT BRITAIN. One Volume, 434 pages octavo; bound in cloth, \$1.50; in goat, \$2.00.
- III. GERMAN EXPERIENCE IN THE ORGANIZATION, INSTRUCTION, AND DISCIPLINE OF PUBLIC OR COMMON SCHOOLS; WITH TREATISES ON PEDAGOGY, DIDACTICS, AND METHODOLOGY. By Professor Raumer, Dr. Diesterweg, Dr. Hentschel, Dr. Abbenrode, Dr. Dinter, and others. One Volume, 520 pages octavo; bound in cloth, \$2.00.
- IV. EDUCATIONAL APHORISMS AND SUGGESTIONS, ANCIENT AND MODERN. One Volume, 200 pages octavo; bound in cloth, \$1.50.
- V. PESTALOZZI AND PESTALOZZIANISM, with sketches of the Educational Views of other Swiss Educators. One Volume, 480 pages octavo; bound in cloth, \$2.00; in goat, with Portrait, \$2.50.
- The above volumes will soon be followed by
- VI. FRENCH AND BELGIAN METHODS OF SCHOOL ORGANIZATION AND INSTRUCTION.
- VII. DUTCH SCHOOLS,—their Organization, Inspection, and Teaching.
- VIII. THE EDUCATION OF GIRLS, with an Account of several of the best Schools for Girls in different Countries.
- IX. SECONDARY EDUCATION; or Subjects and Methods of Instruction in Gymnasias, Lyceas, Grammar Schools, Academies, and High Schools in different Countries. \$2.00.

TERMS.

Any volume will be sent by mail at the above prices.
Orders will be received for the series, in cloth, at \$ 1.50 each.
The volumes will average over 400 octavo pages each, in long
primer type, and in neat cloth binding.

F. C. BROWNELL,
25 Howard Street, New York City.

GEORGE SHERWOOD,
122 & 124 Lake Street, Chicago, Ill.

FOR SALE.

Barnard's American Teachers and Educators, with 21 Portraits, in cloth,	\$3.00.
" Benefactors of American Education, with 12 Portraits, . . .	3.00.
" Reformatory Education and Schools, paper, . . .	1.50.
" Tribute to Gallaudet, cloth, . . .	1.50.
" Raumer's German Universities, cloth, \$1.50; goat, . . .	2.00.
" Normal Schools (<i>new edition</i>), . . .	2.00.
" School Architecture, . . .	2.00.
" National Education in Europe, . . .	3.00.
American Journal of Education, single number, . . .	1.50.
" " " volume, in cloth, . . .	3.00.
" " " First Series, Vols. I.-V., . . .	12.50.
" " " Second Series, Vols. VI.-X., . . .	12.50.

✍ Subscriptions received for Barnard's American Journal of Education, \$4.00 per year, (viz. four numbers of 304 8vo pages each,) payable in advance.

☛ Catalogues containing contents, &c., will be sent on application.

A MONTHLY JOURNAL OF

School and Home Education.

Prospectus of Vol. II., New Series.

THE influence of the TEACHER in carrying forward many salutary reforms, and furnishing a medium of communication and a bond of union among teachers, cannot be doubted by any one familiar with the educational movements of the last ten years. It has been received with favor abroad, and is cherished with affection at home, by those who have taken the lead in the educational reforms of the day. The TEACHER is, as it has ever been, the

OFFICIAL ORGAN OF THE STATE TEACHERS' ASSOCIATION,
and of the *Department of Public Instruction*. It has also been adopted by the
Association of School Officers.

The same general features will characterize it as heretofore; and it is hoped that, with a better organization of our editorial force, we may be able to merit a continuance of our present patronage and to enlist many new friends. The following is the general range of the subjects to be presented in the new volume:—

1. A LEADING ARTICLE BY THE EDITOR.
2. ESSAYS ON SCHOOL STUDIES.
3. HOME EDUCATION.
4. GENERAL LITERATURE.
5. REVIEWS AND NOTICES OF NEW BOOKS.
6. REPORTS OF EDUCATIONAL MEETINGS.
7. SCIENCE AND ART INTELLIGENCE.
8. MISCELLANY.

Each number will contain 48 large octavo pages, making a volume of 576 pages annually, at the small price of

ONE DOLLAR A YEAR.

All new subscriptions commence with the volume. We can supply a few copies of Vol. I, New Series, to our new subscribers, at 80 cents. *Payments to be made strictly in advance.*

Address, enclosing cash,

JAMES CRUIKSHANK.

Editor and Publisher, Albany, N. Y.

DECEMBER, 1860.

AND

School Officer.

A Monthly of Thirty-two royal octavo Pages.

Edited by HON. E. P. WESTON, Superintendent of Common Schools.

THE MAINE TEACHER has had an existence of two and a half years, and will commence the New Year with an edition of 3,000 copies; it is the only educational periodical in the State, and has secured the services of editors and contributors who will not fail to make it a *readable magazine*, not a mere storehouse of green and heavy lumber.

It will aim to be a living and efficient power among our educational forces. In addition to its contributed essays and discussions, each number will contain a *Mathematical Department*, a chapter for School Officers, notices of educational movements in the State and elsewhere, of new publications, and of whatever will be judged most interesting to its readers.

PROPOSALS.

I hereby authorize any teacher or school-officer, or any person whom the school committee will recommend, to act as agent for the MAINE TEACHER within the town where he resides.

For every ten subscribers, with the money for one year, I will give three dollars' worth of choice books at publishers' prices, to be selected from a list which I will publish next month, and for additional subscribers in proportion. I will give, in addition to this amount of books, to the one person in each county who shall send us, with the money, the largest number of subscribers greater than ten, a copy of either Webster's or Worcester's Pictorial Dictionary unabridged.

Agents will have two months between this time and the first of February, 1861, to canvass their respective towns, and may send in names as they obtain them. The names of the persons receiving the county premiums, with their towns and the number on their respective lists, will be published in the February number.

TERMS,-- except as proposed above.

One copy, one year, \$ 1.00	Ten copies, one year, \$ 8.00
Six copies, one year, 5.00	Fifteen copies, one year, . . . 12.00
Twenty copies, one year, \$ 15.00.	

ADVERTISING.

The Maine Teacher affords an invaluable medium of advertising for Publishers, Manufacturers of School Furniture, and other articles of trade. All communications concerning the Teacher must be addressed to

B. THURSTON, Publisher.

PORTLAND, December, 1860.

Portland, Maine.

GROVER & BAKER'S

Celebrated

SEWING MACHINES,

FOR

Family Use and Manufacturing Purposes.

OVER
40,000
IN DAILY USE

IN THE

United States



AND
Territories,
GIVING
UNIVERSAL
SATISFACTION.

THESE MACHINES sew from two spools; the thread requires no re-winding; and they make a STRONGER, HANDSOMER, AND MORE ELASTIC SEAM than any other Machine. They perform their work with a rapidity never before equalled, are almost noiseless in their operation, and are FAVORITES in every family where they have been introduced.

The "Scientific American," November 5th, 1859, *is considered the most useful article in the house, next to the cradle, and no less indispensable than that.*
says:—

"The machines manufactured by the Grover & Baker Sewing Machine Company are too well known by the public at large to need any recommendations at our hands, and we will simply add, that we have had one of them in use in our family for some time past, and it

"Zion's Herald," Nov. 16th, 1859, says:—

"We would recommend to families and to those about to organize a homestead, to be sure and have a Family Bible, and also one of Grover & Baker's SEWING MACHINES."

GROVER & BAKER SEWING MACHINE CO.,

18 Summer Street, Boston.
495 Broadway, New York.
730 Chestnut Street, Philadelphia.
181 Baltimore Street, Baltimore.

58 West Fourth Street, Cincinnati.
115 Lake Street, Chicago.
124 North Fourth Street, St. Louis.
91 Montgomery Street, San Francisco.

THE
MATHEMATICAL MONTHLY.

Vol. III . . . DECEMBER, 1860. . . No. III.

PRIZE PROBLEMS FOR STUDENTS.

I. FROM AC , the diagonal of a square $ABCD$, cut off AE equal to one fourth of AC , and join BE , DE . Show that the figure $BADE$ equals twice the square on AE .

II. Two toothed wheels work against each other. Show that, if the number of teeth in one be prime to that in the other, before two teeth, which have once been in contact, come in contact again, every tooth of the one wheel will have been in contact with every tooth of the other.

III. Eliminate φ from the equations

$$\begin{aligned}x \cos (\varphi + \alpha) + y \sin (\varphi + \alpha) &= a \sin 2 \varphi, \\y \cos (\varphi + \alpha) + x \sin (\varphi + \alpha) &= 2 a \cos 2 \varphi,\end{aligned}$$

showing that

$$(x \sin \alpha - y \cos \alpha)^2 + (y \sin \alpha + x \cos \alpha)^2 = (2 a)^2.$$

IV. Show that $\frac{1}{4} m^2$ is the area of the greatest triangle which can be formed with the lines a , b , c , subject to the condition $a^3 + b^3 + c^3 = 3 m^3$.

V. The equation of a family of ellipses is $a x^2 + b y^2 = 1$, when $a - b = c$, a constant. Show that the curve, which cuts all the individuals at an angle whose tangent is $\frac{y}{x}$, is $x = C e^{-\frac{c y^2}{2}}$.

Solutions of these problems must be received by February 1, 1861.

NOTES AND QUERIES.

1. *Perfect Squares.* — It is a well-known fact that every perfect square, when represented by the Arabic or Hindoo method, must end in 0, 1, 4, 5, 6, or 9. This fact enables us to decide, at a glance, that all numbers ending in 2, 3, 7, or 8 are not perfect squares; but it does not aid us in deciding which of the numbers ending in 0, 1, 4, 5, 6, or 9 are perfect squares, for the converse of the proposition stated above is not true. To assist, to some extent, in removing this difficulty, I submit the following discussion.

PROPOSITION. — The last two figures of the square of any integer must correspond to the last two figures of the square of some number in the series 1, 2, . . . 99. The endings of the squares of the first twenty-five numbers form the series 01, 04, 09, 16, 25, 36, 49, 64, 81, 00, 21, 44, 69, 96, 25, 51, 89, 24, 61, 00, 41, 84, 29, 71, 24. This series also represents the endings of the squares of the numbers from 51 to 75, inclusive. The series, taken backwards, represents the endings of the numbers from 25 to 49, and from 75 to 99, inclusive. This shows that the squares of the numbers equally above and below 25 and 75 end in the same two figures. Omitting the like endings in the above series, and arranging in order, we have 00, 01, 04, 09, 16, 21, 24, 25, 29, 36, 41, 44, 49, 56, 61, 64, 69, 76, 81, 84, 89, 96. These endings indicate the following facts: Every perfect square ending in 1, 4, or 9 must have before the last figure one of the even numbers 0, 2, 4, 6, or 8; ending in 6, must be preceded by one of the odd numbers 1, 3, 5, 7, or 9; ending in 5, by 2; and ending in 0, by 0. — Prof. W. D. HENKLE, Normal School, Lebanon, Ohio.

2. *Problem.* — Prove that, when x is a whole number and greater than one, $x^3 - x$ is divisible by 6.

Suppose that $x^3 - x$ is divisible by 6 when $x = p$, then will it be divisible by 6 when $x = p + 1$. For, letting $x = p + 1$, we have

$$x^3 - x = p^3 + 3p^2 + 3p + 1 - (p + 1) = p^3 + 3p^2 + 2p = p^3 - p + 3p^2 + 3p.$$

Now $p^3 - p$ is divisible by 6, by hypothesis; and if we can prove that $3p^2 + 3p$ is also divisible by 6, then it will follow that $x^3 - x$ is divisible by 6 when $x = p + 1$. But to prove that $3p^2 + 3p$ is divisible by 6, we must prove that $p^2 + p$ is divisible by 2. Suppose that $p^2 + p$ is divisible by 2 when $p = r$, then will it be divisible by 2 when $p = r + 1$. For, putting $p = r + 1$, we get

$$p^2 + p = r^2 + 2r + 1 + r + 1 = r^2 + 3r + 2 = r^2 + r + 2r + 2.$$

But $r^2 + r$ is divisible by 2 by hypothesis, and $2r + 2$ is also divisible by 2. Now we know that $p^2 + p$ is divisible by 2 when $p = 1$, \therefore it must be when $p = 2, 3, 4$, &c.; and therefore if $x^3 - x$ is divisible by 6 when $x = p$, it will be when $x = p + 1$. But we know by inspection that it is divisible when $x = 2$; \therefore it will be when $x = 3, 4, 5$, &c. — JOHN SCOTT, Jr., Night High School, Cincinnati, Ohio.

3. *Note on the Prize Solution of Problem III.*, p. 25. — As there has been a slight typographical error each time the formulas have been printed, I send them in their correct and most reduced form. As m and n exchange values in the two solids, the formulas take very nearly the same forms, one having cosine where the other has the square of the cotangent, and 3 in the place of 2. The last forms given below are adapted to logarithmic computation.

$$\text{Vol. Dodec.} = \frac{5 e^3 \cot^2 \beta}{2 \sqrt{(4 \sin^2 \beta - 1)}} = e^3 \left(\frac{15 + 7 \sqrt{5}}{4} \right) = \frac{5 e^3 \tan^2 54^\circ}{4 \sqrt{(\sin 6^\circ \cos 24^\circ)}}.$$

$$\begin{aligned} \text{Vol. Icos.} &= \frac{5 e^3 \cos \beta}{3 \sqrt{(3 - 4 \cos^2 \beta)}} = \frac{5 e^3 \cos \beta}{3 \sqrt{(4 \sin^2 \beta - 1)}} \\ &= e^3 \left(\frac{15 + 5 \sqrt{5}}{12} \right) = \frac{5 e^3 \sin 54^\circ}{6 \sqrt{(\sin 6^\circ \cos 24^\circ)}}. \end{aligned}$$

— PROF. D. W. HOYT, New Hampton Institution, Fairfax, Vt.

4. *Correction of the Method of Computing Six per Cent Interest for Days.*

— The usual method of computing interest at six per cent, for a given

number of days, that is, "Divide the principal by 6,000 and multiply the quotient by the given number of days," is not of course accurate when large numbers are involved; but the following development, p , denoting the principal, and d the days,

$$\frac{0.06 \, p \, d}{365.25} = \frac{p \, d}{6000} \{1 + \frac{1}{70} [1 + \frac{1}{50} (1 + \frac{1}{24})]\},$$

shows that the correction is almost exactly $\frac{1}{70}$ of the first result; and if this is not sufficient, the second correction is $\frac{1}{50}$ of the first. The third correction, $\frac{1}{24}$ of the second, will never be needed, and is not quite so convenient to apply mentally as the others, but it is accurate to 0.000001 of its value. It will be noticed that these corrections are all additive. All except the first, however, are more curious than useful. — B.

5. *Solution of Problem V., No. IX., Vol. II.* — Two great circles are drawn at random on a sphere. What is the probability that their mutual inclination, taken less than 90° , will be contained between any given limits, as n° and m° ?

1st. The mutual inclination of the two circles will be equal to the angular distance of their poles.

2d. The pole of either great circle is just as likely to fall in one position as in any other; hence the probability that it will fall in any given portion of the surface of the sphere is proportional to the superficial area of that portion.

3d. In order that the angular distance of the poles may be between the limits n° and m° , one pole must fall within the zone contained between two small circles at the respective distances of n° and m° from the other pole.

4th. Hence the probability required will be equal to the area of this zone divided by the area of the hemisphere, or to $\cos n^\circ - \cos m^\circ$. LAPLACE, in considering the inclinations of the orbits of the planets

and comets, in the *Théorie Analytique des Probabilités*, fell into the error of supposing that this probability would be equal to $\frac{m^\circ - n^\circ}{90^\circ}$; or that all inclinations would be equally probable. — S. NEWCOMB, Cambridge, Mass.

6. *Note on the Powers of Binomials.* — Find the fifth power of the binomial $\frac{1}{2}x - 3y^2$.

1	+ 5	+ 10	+ 10	+ 5	+ 1,	Coefficients.
$\frac{1}{32}x^5$	$+\frac{1}{16}x^4$	$+\frac{1}{8}x^3$	$+\frac{1}{4}x^2$	$+\frac{1}{2}x$	+ 1,	Power of $\frac{1}{2}x$.
1	- 3y ²	+ 9y ⁴	- 27y ⁶	+ 81y ⁸	- 243y ¹⁰ ,	Powers of -3y ² .

$$\frac{1}{32}x^5 - \frac{1}{16}x^4y^2 + \frac{1}{8}x^3y^4 - \frac{1}{4}x^2y^6 + \frac{1}{2}xy^8 - 243y^{10} = (\frac{1}{2}x - 3y^2)^5.$$

This simple method of writing powers of complicated binomials is sufficiently obvious. Writing the different lines, and then multiplying together the terms which stand in the same column, so divides up the process, that the whole is made plain and easy to the student. — DAVIES'S University Algebra.

7. *The arithmetic mean of any number of positive quantities is greater than the geometric mean ; that is,*

$$\frac{a + b + c + d \dots + k}{n} > (a b c d \dots k)^{\frac{1}{n}}.$$

Let P denote $(a b c d \dots k)^{\frac{1}{n}}$, and Q denote $\frac{a + b + c + d \dots + k}{n}$. Suppose a and b respectively the greatest and least of the n quantities $a, b, c, d, \dots k$; and let $a_1 = b_1 = \frac{1}{2}(a + b)$, and let $P_1 = (a_1 b_1 c d \dots k)^{\frac{1}{n}}$; then, since $a_1 b_1 > a b$, we have $P_1 > P$. Next, if the factors in P_1 be not all equal, remove the greatest and least of them, and put in their places two new factors, each equal to half the sum of those removed; let P_2 denote the new geometrical mean; then $P_2 > P_1$. If we proceed in this way, we obtain a series, $P, P_1, P_2, P_3, \dots P_r$, each term of which is greater than the preceding term; and by

taking r large enough, we may have the factors of P , as nearly equal as we please; thus, when r is large enough, we may consider $P = Q$, therefore P is less than Q . — TODHUNTER'S Algebra.

8. *Note on the Successive Derivatives of $\tan \varphi$.* — We have

$$D \tan \varphi = D \frac{\sin \varphi}{\cos \varphi} = \frac{\cos^2 \varphi + \sin^2 \varphi}{\cos^2 \varphi} = \frac{1}{\cos^2 \varphi}.$$

$$D^2 \tan \varphi = -\frac{2 D \cos \varphi}{\cos^3 \varphi} = \frac{2 \sin \varphi}{\cos^3 \varphi} = \frac{2 \tan \varphi}{\cos^2 \varphi} = 2 \tan \varphi D \tan \varphi.$$

For brevity, let us adopt the notation $\tan \varphi = D^0$, $D \tan \varphi = D^1$, $D^2 \tan \varphi = D^2$, &c. Then

$$D^2 \tan \varphi = 2 D^1 D^0,$$

$$D^3 \tan \varphi = 2 D^2 D^0 + 2 (D^1)^2,$$

$$D^4 \tan \varphi = 2 D^3 D^0 + 2 D^2 D^1 + 4 D^2 D^1 = 2 D^3 D^0 + 6 D^2 D^1,$$

$$\begin{aligned} D^5 \tan \varphi &= 2 D^4 D^0 + 2 D^3 D^1 + 6 D^3 D^1 + 6 (D^2)^2 \\ &= 2 D^4 D^0 + 8 D^3 D^1 + 6 (D^2)^2, \end{aligned}$$

$$\begin{aligned} D^6 \tan \varphi &= 2 D^5 D^0 + 2 D^4 D^1 + 8 D^4 D^1 + 8 D^3 D^2 + 12 D^3 D^2 \\ &= 2 D^5 D^0 + 10 D^4 D^1 + 20 D^3 D^2. \end{aligned}$$

In general,

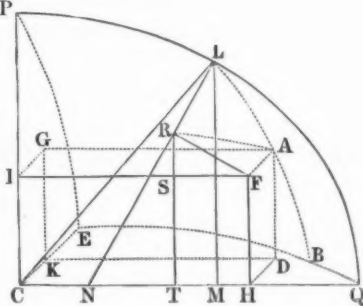
$$\begin{aligned} D^n \tan \varphi &= (D^1 + D^0)^{n-1} \\ &= D^{n-1} D^0 + (n-1) D^{n-2} D^1 + \frac{(n-1)(n-2)}{1 \cdot 2} D^{n-3} D^2 + \frac{(n-1)(n-2)(n-3)}{1 \cdot 2 \cdot 3} D^{n-4} D^3 + \&c. \end{aligned}$$

In this series, terms equally distant from the extremes are equal, and may be added together. These very curious and interesting forms are found in PEIRCE'S *Curves and Functions*, Vol. I.

THE EARTH CONSIDERED AS A SPHEROID OF REVOLUTION.—GEODETIC FORMULAS.

By DR. R. C. MATTHEWSON, U. S. Deputy Surveyor, San Francisco.

LET $CEPQ$ represent one eighth of the spheroid, bounded by the convex surface EPQ , the plane of the equator EQC , and any other two planes, CPE and CPQ , at right angles to each other, passing through the pole P , and the centre C . On the plane CPQ ,* from any point L in the meridian PQ , draw the radius LC , the normal LN , and the ordinate LM . Let the meridional plane CPQ be intersected in L by the normal plane $LN B$, making with it any angle BLQ , BL being the intersection with the surface. Then the curve BL , as well as PQ , will be the arc of an ellipse. In



BL take any point A , and from it let fall the perpendiculars AD , AF , and AG respectively upon the planes CQE , CPE , and CPQ , and complete the parallelograms AH , AI , HK , and IK . On the plane CPQ draw FR at right angles to LN , and RT at right angles to CQ , intersecting FI in S , and join AR .

Put $CM = x$ and $LM = y$, the rectangular co-ordinates of the point L of the meridional ellipse PQ , C being the origin; $NR = x'$ and $AR = y'$, the rectangular co-ordinates of the point A of the normal ellipse BL , N being the origin; $AD = x''$, $AF = y''$, and $AG = z''$, the rectangular co-ordinates of the point A of the spheroidal surface EPQ , C being the origin; $CQ = a$, the equatorial

* For sake of distinction, the lines upon the plane CPQ are continuous, while those not upon it are dotted.

radius ; $CP = b = a(1 - e^2)^{\frac{1}{2}}$, the semi-axis, e being the eccentricity ; $LN = n$, the normal ; $MN = s$, the subnormal ; $CN = m$, the adnormal ; $CL = r$, the radius of the spheroid at the point L ; the angle $LN M = l$, the geographical latitude of the point L ; the angle $LC M = l$, the geocentric latitude of the same point and the angle $AL Q = z$, the azimuth of the point A from L .

Then the equation of the meridional ellipse PQ is

$$a^2 y^2 + b^2 x^2 = a^2 b^2,$$

which, by eliminating b , becomes $y^2 = (1 - e^2)(a^2 - x^2)$. Differentiating this equation, we find the length of the subnormal,

$$s = y \frac{dy}{dx} = (1 - e^2) x = n \cos l.$$

Hence $x = \frac{n \cos l}{1 - e^2}$, and $y = n \sin l$.

Substituting these values for x and y in the equation of the meridional ellipse, we have the normal, $n = \frac{a(1 - e^2)}{(1 - e^2 \sin^2 l)^{\frac{1}{2}}}$; and consequently the abscissa, $x = \frac{a \cos l}{(1 - e^2 \sin^2 l)^{\frac{1}{2}}}$; the ordinate, $y = \frac{a(1 - e^2) \sin l}{(1 - e^2 \sin^2 l)^{\frac{1}{2}}}$;

the subnormal, $s = (1 - e^2) x = \frac{a(1 - e^2) \cos l}{(1 - e^2 \sin^2 l)^{\frac{1}{2}}}$;

the adnormal, $m = x - (1 - e^2) x = e^2 x = \frac{a e^2 \cos l}{(1 - e^2 \sin^2 l)^{\frac{1}{2}}}$;

the radius, $r = (x^2 + y^2)^{\frac{1}{2}} = a \left(1 - \frac{e^2(1 - e^2) \sin^2 l}{1 - e^2 \sin^2 l} \right)^{\frac{1}{2}}$;

and the geocentric latitude,

$$l' = \frac{s}{x} \tan l = \frac{(1 - e^2)x}{x} \tan l = (1 - e^2) \tan l.$$

Again, the equation of the spheroidal surface EPQ is

$$a^2 x'^2 + b^2 y'^2 + b^2 z'^2 = a^2 b^2,$$

or, by eliminating b , $x'^2 + (1 - e^2)(y'^2 + z'^2 - a^2) = 0$. Now

in the right-angled triangle AFR we have the hypotenuse $AR = y'$, and the angle $ARF = z$, whence $AF = y' \sin z$ and $FR = y' \cos z$; in the right-angled triangle FRS we have the hypotenuse $FR = y' \cos z$ and the angle $FRS = l$, whence $FS = y' \sin l \cos z$ and $RS = y' \cos l \cos z$; and in the right-angled triangle NRT we have the hypotenuse $NR = x'$ and the angle $RNT = l$, whence $RT = x' \sin l$ and $NT = x' \cos l$. Hence,

$$x'' = AD = IC = RT - RS = x' \sin l - y' \cos l \cos z;$$

$$y'' = AF = CK = y' \sin z;$$

$$z'' = AG = CH = FS + NT + CN = y' \sin l \cos z + x' \cos l + m.$$

Now if we substitute the foregoing values for x'' , y'' , and z'' in the equation of the spheroidal surface EPQ , we shall find the equation of the normal ellipse BL to be

$$Ax'^2 + By'^2 + Cx'y' + Dx' + Ey' + F = 0,$$

in which $A = 1 - e^2 \cos^2 l$; $B = 1 - e^2 + e^2 \cos^2 l \cos^2 z$;

$$C = -2e^2 \sin l \cos l \cos z; \quad D = \frac{2ae^2(1-e^2)\cos^2 l}{(1-e^2\sin^2 l)^{\frac{1}{2}}};$$

$$E = \frac{2ae^2(1-e^2)\sin l \cos l \cos z}{(1-e^2\sin^2 l)^{\frac{1}{2}}}; \quad \text{and} \quad F = -(1-e^2)(a^2 - m^2).$$

Differentiating the equation of the normal ellipse BL twice, and, in each of the differential coefficients, making $x' = n = \frac{a(1-e^2)}{(1-e^2\sin^2 l)^{\frac{1}{2}}}$ and $y' = 0$, the values of the co-ordinates when the point A coincides with L , we have

$$\frac{dx'}{dy'} = 0, \quad \text{and} \quad \frac{d^2x'}{dy'^2} = -\frac{2D}{2Ax' + D}.$$

Substituting these values for $\frac{dx'}{dy'}$, and $\frac{d^2x'}{dy'^2}$ in the general expression for the radius of the osculating circle, and then substituting the above values of A , B , D , and x' in the result, we have

$$\begin{aligned}
 R_z &= -\frac{\left(1 + \frac{d^2 x^2}{d y^2}\right)^{\frac{1}{2}}}{\frac{d^2 x^2}{d y^2}} = \frac{A x' + \frac{1}{2} D}{B} = \frac{a(1-e^2)}{(1-e^2 + e^2 \cos^2 l \cos^2 z)(1-e^2 \sin^2 l)^{\frac{1}{2}}} \\
 &= \frac{a(1-e^2)}{(1-e^2 \sin^2 z - e^2 \sin^2 l \cos^2 z)(1-e^2 \sin^2 l)^{\frac{1}{2}}} \\
 &= \frac{a(1-e^2)}{(\sin^2 z + \cos^2 z - e^2 \sin^2 z - e^2 \sin^2 l \cos^2 z)(1-e^2 \sin^2 l)^{\frac{1}{2}}} \\
 &= \frac{a(1-e^2)}{(1-e^2)(1-e^2 \sin^2 l)^{\frac{1}{2}} \sin^2 z + (1-e^2 \sin^2 l)^{\frac{1}{2}} \cos^2 z},
 \end{aligned}$$

which is the general expression for the radius of curvature of any normal ellipse on the spheroid making, at any latitude l , any angle z with the meridian.

When the normal ellipse BL coincides with the meridian, the angle $z = 0$, $\sin z = 0$, $\cos z = 1$, and R_z becomes $R_m = \frac{a(1-e^2)}{(1-e^2 \sin^2 l)^{\frac{1}{2}}}$, which is the radius of curvature of the meridian at the latitude l . When the normal ellipse BL is at right angles to the meridian, the angle $z = 90^\circ$, $\sin z = 1$, $\cos z = 0$, and R_z becomes $R_n = \frac{a}{(1-e^2 \sin^2 l)^{\frac{1}{2}}}$, which is the radius of curvature of the perpendicular to the meridian at the latitude l . If $l = 0$, $\sin l = 0$, and we have $R_m = a(1-e^2)$ and $R_n = a$. If $l = 90^\circ$, $\sin l = 1$, and we have $R_m = R_n = \frac{a}{(1-e^2)^{\frac{1}{2}}}$. In the former case the normal ellipse BL coincides with the equator, while in the latter it coincides with the meridian at the pole.

The radius of curvature of any normal ellipse on the spheroid, whether coinciding with the plane of the meridian, at right angles, or oblique to it, may also be obtained in the following manner. The radius of curvature of the meridian may be derived, at once, from the equation of the meridional ellipse $y^2 = (1-e^2)(a^2 - x^2)$, by substituting the values of the first and second differential coefficients in the general expression for the radius of the osculating circle, and making the proper substitutions and eliminations. It will be found that

$$\frac{d y^2}{d x^2} = -\frac{(1-e^2)x}{a^2 - x^2}, \quad \text{and} \quad \frac{d^2 y}{d x^2} = -\frac{a^2(1-e^2)^{\frac{1}{2}}}{(a^2 - x^2)^{\frac{3}{2}}},$$

whence
$$R_m = \frac{(a^2 - e^2 x^2)^{\frac{1}{2}}}{a^2 (1 - e^2)} = \frac{a (1 - e^2)}{(1 - e^2 \sin^2 l)^{\frac{1}{2}}}.$$

The radius of curvature of the perpendicular to the meridian is the normal LN produced to the axis of the spheroid, whence it is evident that $MN : CM :: LN : R_n$, or $S : x :: n : R_n$ and consequently

$$R_n = \frac{nx}{S} = \frac{nx}{(1 - e^2)x} = \frac{n}{1 - e^2} = \frac{a}{(1 - e^2 \sin^2 l)^{\frac{1}{2}}}.$$

It can then be easily proved that

$$R_z = \frac{R_m R_n}{R_m \sin^2 z + R_n \cos^2 z}^* = \frac{a (1 - e^2)}{(1 - e^2)(1 - e^2 \sin^2 l)^{\frac{1}{2}} \sin^2 z + (1 - e^2 \sin^2 l)^{\frac{1}{2}} \cos^2 z}.$$

In order to find the length of a small arc δ° of a normal ellipse, in any position, on the surface of the spheroid, we have merely to multiply the radius of curvature at l , the middle latitude of δ° , by π , the ratio of the semi-circumference to the radius of a circle, and divide the product by $\frac{180^\circ}{\delta^\circ}$. If $\delta = 1^\circ$, we have

$$D_n = \frac{\pi}{180} R_n = \frac{\pi}{180} \cdot \frac{a}{(1 - e^2 \sin^2 l)^{\frac{1}{2}}} = \frac{\pi a}{180} (1 - e^2 \sin^2 l)^{-\frac{1}{2}},$$

which is the length of an arc of one degree of the perpendicular to the meridian at the mean latitude l ;

$$D_m = \frac{\pi}{180} R_m = \frac{\pi}{180} \cdot \frac{a (1 - e^2)}{(1 - e^2 \sin^2 l)^{\frac{1}{2}}} = \frac{\pi a (1 - e^2)}{180} (1 - e^2 \sin^2 l)^{-\frac{1}{2}},$$

which is the length of an arc of one degree of the meridian at the mean latitude l ; and

$$D_z = \frac{\pi}{180} R_z = \frac{\pi}{180} \frac{R_m R_n}{R_m \sin^2 z + R_n \cos^2 z},$$

which is the length of an arc of one degree making any angle z with the meridian, at the mean latitude l .

* This Formula is wrong in Capt. T. J. LEE's Tables and Formulæ, Article XI. p. 34.

If, in the last expressions for the length of a degree of the perpendicular and meridian, we develop the second factors by the binomial theorem, we have

$$D_n = \frac{\pi a}{180} (1 + \frac{1}{2} e^2 \sin^2 l + \frac{3}{8} e^4 \sin^4 l + \frac{5}{16} e^6 \sin^6 l + \frac{35}{128} e^8 \sin^8 l + \&c.),$$

and

$$D_m = \frac{\pi a (1 - e^2)}{180} (1 + \frac{3}{2} e^2 \sin^2 l + \frac{15}{8} e^4 \sin^4 l + \frac{35}{16} e^6 \sin^6 l + \frac{315}{128} e^8 \sin^8 l + \&c.).$$

Now $\pi = 3.141592654$, and, according to the data adopted by the Superintendent of the United States Coast Survey, $a = 6377397.156$ metres, and $e = 0.081696830$. Substituting these values for π , a , and e , in the developed expressions for the length of a degree of the perpendicular and meridian, we have, in powers of $\sin l$,

$$(1) \quad D_n = 111306.57808 + 371.45076 \sin^2 l + 1.85940 \sin^4 l \\ + 0.01034 \sin^6 l + 0.0006 \sin^8 l$$

$$(2) \quad D_m = 110563.67657 + 1106.91467 \sin^2 l + 9.23495 \sin^4 l \\ + 0.07191 \sin^6 l + 0.00054 \sin^8 l$$

$$\text{But} \quad \sin^2 l = 1 - \cos^2 l; \quad \sin^4 l = 1 - 2 \cos^2 l + \cos^4 l;$$

$$\sin^6 l = 1 - 3 \cos^2 l + 3 \cos^4 l - \cos^6 l;$$

$$\text{and} \quad \sin^8 l = 1 - 4 \cos^2 l + 6 \cos^4 l - 4 \cos^6 l + \cos^8 l$$

Substituting these values for the different powers of $\sin l$ in (1) and (2), we have, for the length of a degree of the perpendicular and meridian in powers of $\cos l$,

$$(3) \quad D_n = 111679.89864 - 375.20082 \cos^2 l + 1.89079 \cos^4 l \\ - 0.01058 \cos^6 l + 0.00006 \cos^8 l$$

$$(4) \quad D_m = 111679.89864 - 1125.60246 \cos^2 l + 9.45392 \cos^4 l \\ - 0.07407 \cos^6 l + 0.00054 \cos^8 l$$

$$\text{Now} \quad \sin^2 l = \frac{1}{2} - \frac{1}{2} \cos 2l; \quad \sin^4 l = \frac{3}{8} - \frac{1}{2} \cos 2l + \frac{1}{8} \cos 4l;$$

$$\sin^6 l = \frac{5}{16} - \frac{15}{8} \cos 2l + \frac{3}{8} \cos 4l - \frac{1}{16} \cos 6l;$$

and $\sin^8 l = \frac{35}{128} - \frac{7}{16} \cos 2l + \frac{7}{32} \cos 4l - \frac{1}{16} \cos 6l + \frac{1}{128} \cos 8l$. Substituting these values of the different powers of $\sin l$ in (1) and (2), or changing all the negative into positive signs, which changes the values to those of the powers of $\cos l$ instead of $\sin l$, and substituting in (3) and (4), we have, for the length of a degree of the perpendicular and meridian in multiples of $\cos l$,

$$(5) \quad D_n = 111493.00398 - 186.65995 \cos 2l + 0.23438 \cos 4l - 0.00033 \cos 6l$$

$$(6) \quad D_m = 111120.61963 - 558.10875 \cos 2l + 1.16797 \cos 4l - 0.00228 \cos 6l^*$$

If a perpendicular be let fall from the point L in the meridian PQ , upon the semi-axis CP , it will be the radius of the parallel of latitude at that point, and will obviously be equal to the abscissa CM . Hence we have $R_p = R_n \cos l$, the radius of the parallel of latitude, and consequently $D_p = D_n \cos l$, the length of a degree of the parallel at the latitude l . Therefore, multiplying (3) by $\cos l$, we have, for the length of a degree of the parallel at the latitude l in powers of $\cos l$,

$$(7) \quad D_p = 111675.89864 \cos l - 375.20082 \cos^3 l + 1.89079 \cos^5 l - 0.01058 \cos^7 l$$

But $\cos^3 l = \frac{3}{4} \cos l + \frac{1}{4} \cos 3l$; $\cos^5 l = \frac{5}{8} \cos l + \frac{5}{16} \cos 3l + \frac{1}{16} \cos 5l$; and $\cos^7 l = \frac{35}{64} \cos l + \frac{21}{16} \cos 3l + \frac{7}{16} \cos 5l + \frac{1}{64} \cos 7l$. Substituting these values for the different powers of $\cos l$ in (7), we have, for the length of a degree of the parallel at the latitude l in multiples of $\cos l$,

$$(8) \quad D_p = 111399.67397 \cos l - 93.21281 \cos 3l + 0.11702 \cos 5l - 0.00017 \cos 7l$$

In practice it will be necessary to retain only three decimal figures in these formulas, and it will be found more convenient to use the powers when we work by the logarithmic, and the multiples when we work by the natural cosines. The following are the formulas for

* The decimal part of the coefficient of $\cos 2l$ in this formula is wrong in the Report of the Superintendent of the United States Coast Survey, for the year 1853, p. 100.*

the powers and the multiples in French metres, American yards, and English yards.

I. IN POWERS OF $\cos l$.

$$D_m = \begin{cases} 111679.899 - 1125.602 \cos^2 l + 9.454 \cos^4 l - 0.074 \cos^6 l, \text{ in F. metres.} \\ 122129.741 - 1230.925 \cos^2 l + 10.339 \cos^4 l - 0.081 \cos^6 l, \text{ in A. yards.} \\ 122136.829 - 1230.996 \cos^2 l + 10.339 \cos^4 l - 0.081 \cos^6 l, \text{ in E. yards.} \end{cases}$$

$$D_n = \begin{cases} 111679.899 - 375.201 \cos^2 l + 1.891 \cos^4 l - 0.011 \cos^6 l, \text{ in F. metres.} \\ 122129.741 - 410.308 \cos^2 l + 2.068 \cos^4 l - 0.012 \cos^6 l, \text{ in A. yards.} \\ 122136.829 - 410.332 \cos^2 l + 2.068 \cos^4 l - 0.012 \cos^6 l, \text{ in E. yards.} \end{cases}$$

$$D_p = \begin{cases} 111679.899 \cos l - 375.201 \cos^3 l + 1.891 \cos^5 l - 0.011 \cos^7 l, \text{ in F. metres.} \\ 122129.741 \cos l - 410.308 \cos^3 l + 2.068 \cos^5 l - 0.012 \cos^7 l, \text{ in A. yards.} \\ 122136.829 \cos l - 410.332 \cos^3 l + 2.068 \cos^5 l - 0.012 \cos^7 l, \text{ in E. yards.} \end{cases}$$

II. IN MULTIPLES OF $\cos l$.

$$D_m = \begin{cases} 111120.620 - 558.109 \cos 2l + 1.168 \cos 4l - 0.002 \cos 6l, \text{ in F. metr.} \\ 121518.130 - 610.331 \cos 2l + 1.277 \cos 4l - 0.002 \cos 6l, \text{ in A. yds.} \\ 121525.183 - 610.366 \cos 2l + 1.277 \cos 4l - 0.002 \cos 6l, \text{ in E. yds.} \end{cases}$$

$$D_n = \begin{cases} 111493.004 - 186.660 \cos 2l + 0.234 \cos 4l, \text{ in French metres.} \\ 121925.359 - 204.126 \cos 2l + 0.256 \cos 4l, \text{ in American yards.} \\ 121932.434 - 204.137 \cos 2l + 0.256 \cos 4l, \text{ in English yards.} \end{cases}$$

$$D_p = \begin{cases} 111399.674 \cos l - 93.213 \cos 3l + 0.117 \cos 5l, \text{ in French metres.} \\ 121823.296 \cos l - 101.935 \cos 3l + 0.128 \cos 5l, \text{ in American yards.} \\ 121830.366 \cos l - 101.941 \cos 3l + 0.128 \cos 5l, \text{ in English yards.} \end{cases}$$

ON SPHERICAL ANALYSIS.

By GEORGE EASTWOOD, Saxonville, Mass.

[Continued from Page 304, Vol. II.]

PROPOSITION VI.

To find the intersection of two given great circles of the sphere.

In this proposition it is manifest that the abscissa, X , and the ordinate, Y , of the point of intersection are common to both the given circles. If therefore we define the equation of one of them by

$y = \tau x + \beta$, the equation of the other by $y = \tau' x + \beta'$, and the equation of their common point of intersection by

$$Y = \tau X + \beta = \tau' X + \beta',$$

we shall have $X = \frac{\beta' - \beta}{\tau - \tau'}$,

and $Y = \tau X + \beta = \frac{\tau \beta' - \tau' \beta}{\tau - \tau'} + \beta = \frac{\tau \beta' - \tau' \beta}{\tau - \tau'}$,

for the point XY of intersection.

PROPOSITION VII.

To find the intersection of a great with a less circle of the sphere.

Let the centre of the less circle be taken for the origin of co-ordinates; and let, as in Cor. 2 of Prop. II., its equation be represented by

$$y^2 + 2xy \cos \omega + x^2 = r^2.$$

If we solve this equation for y , we shall find

$$y = (r^2 - x^2 \sin^2 \omega)^{\frac{1}{2}} - x \cos \omega.$$

But by (19) $y = \tau x + \beta$,

$$\therefore r^2 - x^2 \sin^2 \omega = (\tau x + x \cos \omega + \beta)^2,$$

$$\therefore r^2 - \beta^2 = (\tau^2 + 2\tau \cos \omega + 1)x^2 + 2\beta(\tau + \cos \omega)x,$$

$$\therefore x = \frac{(r^2(\tau^2 + 2\tau \cos \omega + 1) - \beta^2 \sin^2 \omega)^{\frac{1}{2}} - \beta(\tau + \cos \omega)}{\tau^2 + 2\tau \cos \omega + 1} = X,$$

$$\therefore y = \tau x + \beta = \frac{\tau(r^2(\tau^2 + 2\tau \cos \omega + 1) - \beta^2 \sin^2 \omega)^{\frac{1}{2}} + \beta(\tau \cos \omega + 1)}{\tau^2 + 2\tau \cos \omega + 1} = Y.$$

Now, that this value of y may be possible, it is necessary that we have

$$r^2 > \frac{\beta^2 \sin^2 \omega}{\tau^2 + 2\tau \cos \omega + 1};$$

and that the two circles may touch each other, or be tangential, it is necessary and sufficient that

$$(27) \quad r = \pm \left(\frac{\beta^2 \sin^2 \omega}{\tau^2 + 2\tau \cos \omega + 1} \right)^{\frac{1}{2}} = \frac{\pm \beta \sin \omega}{(\tau^2 + 2\tau \cos \omega + 1)^{\frac{1}{2}}}.$$

As r , in this case, is equal to the distance between the pole of the less circle and the circumference of the great circle, it follows that (27) makes known by its tangent the distance from the origin of a great circle whose intercepts on the axes of reference are α and β .

If the axes of reference are rectangular, the condition of contact will become $\beta = \pm r(1 + \tau^2)^{\frac{1}{2}}$; so that the equation of a great circle of the sphere tangent to a less circle may be written

$$(28) \quad y = \tau x \pm r(1 + \tau^2)^{\frac{1}{2}}.$$

Equation (28) may be applied to the solution of the following problem:—

A less circle of the sphere has its centre at the right angle of a right spherical triangle; it is required to draw a great circle tangent to it, so that the tangents of the intercepted segments of the sides of the triangle may have a given ratio.

There are evidently two solutions to this problem: for if we seek the intersection of the two circles $y = \tau x \pm r(1 + \tau^2)^{\frac{1}{2}}$, we see at once that x and y cannot be finite for the two equations

$$y = \tau x + r(1 + \tau^2)^{\frac{1}{2}}, \quad y = \tau x - r(1 + \tau^2)^{\frac{1}{2}},$$

when they subsist together. That they may subsist, we must obviously have $x = \infty$; that is, the intersection must be 90° from the pole of the less circle. Hence, in this case, we must have recourse to the latitude of the point of intersection.

Take \bar{y} for the latitude of the point of intersection, then by (5) $y = \frac{\bar{y}}{\cos x}$; so that the equations of the two tangent circles may be written

$$\bar{y} = \tau \sin x + r(1 + \tau^2)^{\frac{1}{2}} \cos x, \quad \bar{y} = \tau \sin x - r(1 + \tau^2)^{\frac{1}{2}} \cos x,$$

from which we readily deduce $\cos x = 0$, or $x = \frac{\pi}{2}$, and $\therefore \bar{y} = \tau$.

As a further illustration of the subject, let P , in the annexed

diagram, be the point of intersection of the two tangent circles, C and D their points of contact with the given circle, and $A O, O B$ the intercepted segments of the sides of the given triangle. Then, because $O P = O E$ is a quadrantal arc of a great circle, and the angle

$$P O C = \angle P O D = \frac{\pi}{2},$$

we have the latitude

$$P E = \angle P O E = \frac{\pi}{2} - \angle D O E = \frac{\pi}{2} - \angle A O C.$$

And, because the angles $A O B, A C O, B C O$ are each right angles, the spherical right triangles $A C O, B C O$ give

$$\tan C O = \tan A O \cdot \cos A O C = \tan B O \cdot \cos B O C = \tan B O \cdot \sin A O C;$$

$$\therefore \cot A O C = \frac{\tan B O}{\tan A O} = \frac{\beta}{-a} = \tau = \tan P E.$$

If $x' y'$ be the co-ordinates of the point of contact C or D , the equation of the tangent $A P$ or $P D$ may be written

$$(29) \quad y y' + x x' = r^2.$$

For, by (28), the equation of the point of contact is

$$y' = \tau x' + r^2 (1 + \tau^2)^{\frac{1}{2}},$$

in which $r = (x^2 + y^2)^{\frac{1}{2}}$.

Resolving therefore for τ , we find

$$\tau = -\frac{x'}{y'} = \frac{-1}{\frac{y'}{x'}} = -\frac{\beta}{a},$$

the substitution of which in (28) gives $y y' + x x' = r^2$.

PROPOSITION VIII.

To find the distance between two given points on the surface of the sphere.

Designating the latitudes of the given points by \bar{y}_1, \bar{y}_2 , the longi-

tudes by x', x'' , and their distance apart by A , we shall have from the fundamental spherical triangle,

$$\begin{aligned}\cos A &= \sin \bar{y}_1 \sin \bar{y}_2 + \cos \bar{y}_1 \cos \bar{y}_2 \cos (x' - x''), \\ &= \frac{\bar{y}_1 \bar{y}_2 + \cos x' \cos x'' + \sin x' \sin x''}{(1 + \bar{y}_1^2)^{\frac{1}{2}} (1 + \bar{y}_2^2)^{\frac{1}{2}}};\end{aligned}$$

in which $\sin \bar{y}_1 \sin \bar{y}_2$ and $\cos \bar{y}_1 \cos \bar{y}_2$ are replaced by their conventional equivalents $\bar{y}_1 \bar{y}_2$ and $(1 + \bar{y}_1^2)^{\frac{1}{2}} (1 + \bar{y}_2^2)^{\frac{1}{2}}$.

If now we introduce, for the geographical co-ordinates \bar{y}_1 and \bar{y}_2 , the geometrical co-ordinates $y' \cos x'$ and $y'' \cos x''$, or their equals, $\frac{y'}{(1 - x'^2)^{\frac{1}{2}}}$ and $\frac{y''}{(1 - x''^2)^{\frac{1}{2}}}$, we shall have

$$(30) \quad \cos A = \frac{y' y'' + x' x'' + 1}{\pm (1 + x'^2 + y'^2)^{\frac{1}{2}} (1 + x''^2 + y''^2)^{\frac{1}{2}}},$$

$$(31) \quad \sin A = \frac{\{(x'' - x')^2 + (y'' - y')^2 + (x' y'' - x'' y')^2\}^{\frac{1}{2}}}{(1 + x'^2 + y'^2)^{\frac{1}{2}} (1 + x''^2 + y''^2)^{\frac{1}{2}}},$$

$$(32) \quad \tan A = \frac{\pm \{(x'' - x')^2 + (y'' - y')^2 + (x' y'' - x'' y')^2\}^{\frac{1}{2}}}{y' y'' + x' x'' + 1}.$$

In these formulas the double signs indicate the distance of the point $x' y'$ from the point diametrically opposite to the point $x'' y''$, as well as from the point $x'' y''$ itself. This will be obvious when it is remembered that two points on the surface of the sphere may be joined by two different arcs of great circles, which, taken together, make up the whole circumference of a great circle.

If in equation (32) we regard A as constant, and x', y' , as variables, we shall have the equation of a less circle of the sphere referred to an external origin, in functions of the co-ordinates x'', y'' , of its pole, and of its radius A .

When two points on the surface of the sphere are at a distance of 90° from each other, the cosine of this distance is zero; we shall therefore have, between their co-ordinates x', y', x'', y'' , the relation

$$(33) \quad y' y'' + x' x'' + 1 = 0.$$

When two great circles intersect each other at right angles, their poles will be 90° apart; hence the relation (33) will hold for the co-ordinates of their poles.

If two great circles are represented by the equations

$$Mx + Ny = P, \quad M'x + N'y = P',$$

the relation (33) will give

$$(34) \quad MM' + NN' + PP' = 0.$$

And, generally, if two great circles be defined by the equations

$$\frac{y}{\beta} + \frac{x}{\alpha} = 1, \quad \frac{y'}{\beta'} + \frac{x'}{\alpha'} = 1;$$

the relation which expresses that they cut each other at right angles will be truly defined by

$$(35) \quad \frac{1}{\beta'\beta} + \frac{1}{\alpha'\alpha} + 1 = 0.$$

PROPOSITION IX.

To find an expression for the angle formed by the intersection of two great circles of the sphere.

The angle formed by the intersection of the two circles is evidently equal to the distance between their poles; \therefore if θ represents this distance, and if $x', y', x'' y''$, be the co-ordinates of the poles, equation (30) will give

$$\cos \theta = \frac{y' y'' + x' x'' + 1}{(1 + x'^2 + y'^2)^{\frac{1}{2}} (1 + x''^2 + y''^2)^{\frac{1}{2}}},$$

for the cosine of the required angle.

But if the circles be represented by $\alpha y + \beta x = \alpha \beta$ and $\alpha' y + \beta' x = \alpha' \beta'$, Prop. IV., gives

$$x' = -\frac{1}{\alpha}, \quad y' = -\frac{1}{\beta} \quad \text{and} \quad x'' = -\frac{1}{\alpha'}, \quad y'' = -\frac{1}{\beta'}.$$

Hence

$$(36) \quad \cos \theta = \frac{\alpha \alpha' + \beta \beta' + \alpha \alpha' \beta \beta'}{(\alpha^2 + \beta^2 + \alpha^2 \beta^2)^{\frac{1}{2}} (\alpha'^2 + \beta'^2 + \alpha'^2 \beta'^2)^{\frac{1}{2}}}.$$

Equation (36) shows that if

$$(37) \quad \alpha \alpha' + \beta \beta' + \alpha \alpha' \beta \beta' = 0,$$

the two circles will be perpendicular to each other.

PROPOSITION X.

To find the distance of a given point on the surface of the sphere from the circumference of a great circle.

Let the given point be designated by $x' y'$, the pole of the given circle by $x'' y''$, the distance between them by Δ , and the required distance by Δ' ; then, by (32),

$$\tan \Delta = \frac{\pm \{(x'' - x')^2 + (y'' - y')^2 + (x' y'' - x'' y')^2\}^{\frac{1}{2}}}{y' y'' + x' x'' + 1}.$$

But $\tan \Delta' = \cot \Delta$, hence

$$(38) \quad \tan \Delta' = \frac{y' y'' + x' x'' + 1}{\pm \{(x'' - x')^2 + (y'' - y')^2 + (x' y'' - x'' y')^2\}^{\frac{1}{2}}},$$

makes known the required distance.

If for x'' and y'' we substitute their values, $-\frac{1}{\alpha'}$ and $-\frac{1}{\beta'}$, then (38) becomes

$$(39) \quad \tan \Delta' = \frac{-\alpha' y' - \beta' x' + \alpha' \beta'}{\pm \{(\alpha' x' - 1)^2 + (\beta' y' - 1)^2 + (\alpha' x' - \beta' y')^2\}^{\frac{1}{2}}},$$

Again, since

$$\sin \Delta = \frac{\tan \Delta'}{(1 + \tan^2 \Delta')^{\frac{1}{2}}} = \frac{-\alpha' y' - \beta' x' + \alpha' \beta'}{\pm \{(\alpha'^2 + \beta'^2 + \alpha'^2 \beta'^2)(1 + x'^2 + y'^2)\}^{\frac{1}{2}}},$$

\therefore in general, if the equation of a great circle be represented by $\mu y + \nu x + \rho = 0$, then a perpendicular upon its circumference from a given point, $x' y'$, may be determined from the equation

$$(40) \quad \sin \Delta = \frac{\mu y' + \nu x' + \rho}{\mp \{(\mu^2 + \nu^2 + \rho^2)(1 + x'^2 + y'^2)\}^{\frac{1}{2}}}.$$

PROPOSITION XI.

To find the equation of a great circle passing through a given point, and perpendicular to another given great circle.

Let the given great circle be defined by the equation

$$(a) \quad \frac{y'}{\beta'} + \frac{x'}{\alpha'} - 1 = 0,$$

and the required one by the equation

$$(b) \quad \frac{y}{\beta} + \frac{x}{\alpha} - 1 = 0.$$

Then, that the conditions of the problem may be fulfilled, we must have, by (35) and (23),

$$(c) \quad \frac{1}{\alpha \alpha'} + \frac{1}{\beta \beta'} + 1 = 0, \quad y - y' = \tau' (x - x') = -\frac{\beta'}{\alpha'} (x - x').$$

Now, from (b) we find

$$(d) \quad \alpha = \frac{\beta x}{\beta - y},$$

and from (a) we find $\alpha' = \frac{\beta' x'}{\beta' - y'};$

$$(e) \quad \therefore \alpha \alpha' = \frac{\beta \beta' x x'}{(\beta - y)(\beta' - y')}.$$

Hence (c) becomes $\frac{(\beta - y)(\beta' - y')}{\beta \beta' x x'} + \frac{1}{\beta \beta'} + 1 = 0,$

which gives $\beta' = \frac{(\beta - y)y' - x'x}{\beta - y + \beta x'x} = \frac{\beta y' - \alpha x'}{(1 + \alpha x')\beta},$

by virtue of (d).

Again, from (e) we have

$$\alpha' = \frac{\beta \beta' x x'}{(\beta - y)(\beta' - y') \alpha} = \frac{\{(\beta - y)y' - x'x\} \beta}{-(\beta - y)(1 + \beta y') \alpha} = \frac{\beta y' - \alpha x'}{-(1 + \beta y') \alpha}.$$

Hence $\tau' = -\frac{\beta'}{\alpha'} = \frac{\alpha(1 + \beta y')}{\beta(1 + \alpha x')}$, and therefore the required equation is

$$(41) \quad y - y' = \frac{\alpha(1 + \beta y')}{\beta(1 + \alpha x')} (x - x').$$

If the co-ordinates of the pole of the given circle be x'' and y'' ,

then, since $\alpha = -\frac{1}{x''}$ and $\beta = -\frac{1}{y''}$, equation (41) gives

$$y - y' = \frac{y' - y''}{x' - x''} (x - x'),$$

which accords with (25), as it evidently ought to do.

A TREATISE ON DETERMINANTS.

CHAPTER I.

INTRODUCTORY.

1. If, by the usual methods of elimination, we solve the linear equations $a'x + b'y = m'$, $a''x + b''y = m''$, we get

$$x = \frac{m'b'' - m''b'}{a'b'' - a''b'}, \quad y = \frac{a'm'' - a''m'}{a'b'' - a''b'};$$

whose numerators and common denominator have each that peculiar "alternate symmetry," whereby either changes merely in sign if we transpose any two letters throughout without moving the indices; e. g. $(m'b'' - m''b') = -(b'm'' - b''m')$. So also, if we transpose two indices throughout without moving the letters; e. g. $(m'b'' - m''b') = -(m''b' - m'b'')$. Indeed the symmetry of either polynomial is precisely the same with respect to its indices as with respect to its letters. Moreover, the numerator of x or of y is formed from the common denominator, by writing m instead of a or instead of b .

All these properties belong as well to any n values x, y , &c., determined by n given linear equations. Thus,

$$\begin{aligned} x &= \frac{m'b''c''' + m''b'''c' + m'''b'c'' - m'''b''c' - m''b'c''' - m'b'''c''}{a'b''c''' + a''b'''c' + a'''b'c'' - a'''b''c' - a''b'c''' - a'b'''c''}, \\ y &= \frac{a'm''c''' + a''m'''c' + a'''m'c'' - a'''m''c' - a''m'c''' - a'm'''c''}{a'b''c''' + a''b'''c' + a'''b'c'' - a'''b''c' - a''b'c''' - a'b'''c''}, \\ z &= \frac{a'b''m''' + a''b'''m' + a'''b'm'' - a'''b''m' - a''b'm''' - a'b'''m''}{a'b''c''' + a''b'''c' + a'''b'c'' - a'''b''c' - a''b'c''' - a'b'''c''}, \end{aligned}$$

which evidently satisfy the equations

$$a'x + b'y + c'z = m', \quad a''x + b''y + c''z = m'', \quad a'''x + b'''y + c'''z = m''',$$

have these properties.

Again, the numerators and denominator of the set of values obtained from n equations may be found, by a simple law, from the values that satisfy sets of $(n-1)$ equations. Thus the above common denominator of x, y, z , may be written

$$\begin{aligned} & a'(b''c''' - b'''c'') + a''(b'''c' - b'c''') + a'''(b'c'' - b''c'), \\ \text{or} \quad & a'(b''c''' - c''b''') + b'(c''a''' - a''c''') + c'(a'b''' - b''a'''); \end{aligned}$$

whose parentheses are the denominators of those values that satisfy the equations

$$b'\xi + c'\eta = m', \quad b''\xi + c''\eta = m'', \quad b'''\xi + c'''\eta = m''';$$

or else the equations

$$a'\xi + a''\eta = A, \quad b'\xi + b''\eta = B, \quad c'\xi + c''\eta = C,$$

taken two and two.

When the common denominator vanishes, some or all of x, y, \dots must become infinite, indicating that the given equations are incompatible; unless all m', m'', \dots likewise vanish, in which case x, y, \dots are indeterminate, indicating that some of the given equations are deducible from the rest.

2. These numerators and denominators are called DETERMINANTS. Their symmetry should appear in most investigations where systems of linear equations enter; and their vanishing commonly indicates that certain co-efficients or functions are not independent. Their properties will often aid us when the difficult or characteristic feature of a problem is to eliminate among the equations or to combine the results of elimination.

The subject of Derivatives assumes a linear character, [we may

say, through the systematic omission of the higher infinitesimals,] such that the change of systems of independent variables in differentiating or integrating, the assignment of multiple definite integrals, and the properties of linear differential equations, so important in dynamics, involve determinants.

In Algebra, equations of any degree have been treated as linear equations, whose new variables are powers and products of the old ones, and may be eliminated by determinants. By such elimination among equations derived from a single one by differentiation, we obtain determinants; e. g., the “discriminant” whose vanishing shall express certain equalities among the roots, and which are “invariants.” The extensive theory of invariants, &c., has also other connections with determinants.

In Geometry, we naturally work by combining right lines and planes, whose equations are linear. Co-ordinates are often changed by “linear substitution,” as in Perspective, and this introduces Invariants, &c. The contents of triangles and pyramids, being definite integrals, are expressed by determinants.

In the Theory of Numbers and elsewhere, it is natural that those determinants whose vanishing changes the character of any problem, should likewise often affect it by their form or smallness when they do *not* vanish. Especially, the discriminant of a quadratic form so *determines* the character of that form, as to have first received from GAUSS the name “determinant.” (Disq. Arith. § ...)

The preference of linear forms, even in Geometry and by Nature in Dynamics, appears to be not merely because linear equations are the simpler, as quadratics are simpler than cubics; but also because, unlike all higher equations, they give but one value to each variable, which is always real if the co-efficients and absolute terms are so, the degree, too, being never raised by eliminating any of the variables.

3. Though regarded chiefly as a rather simple instrument of investigation, Determinants have also an interest of their own. I shall present what I suppose to be their leading features and their general modes of application.*

What follows will be chiefly from other treatises; most from BRIOSCHI, but much also from SALMON, PEIRCE, FAA DE BRUNO, SPOTTISWOODE, and from several memoirs of CAYLEY and others; with occasional suggestions of my own. Writing rather of the theory than of its authors, I could not always mention the latter without more reading, neither do I care to keep their work unchanged.

I shall try to state clearly, in coarse type, what is at first most important; giving more concisely, and in finer print, things which the reader may omit at his option, as the fine print will never be needed to demonstrate anything that follows in coarse. But he would best notice any *examples* in small type which follow and illustrate the large.

Whether for mere *reference*, or to indicate final *results*, formulas, and perhaps verbal statements, will be numbered in one series, thus, (1), (2), ... Some important results may be marked by bold figures, thus, (2). Analogous statements may be indicated by accents, as, (2), (2'), (2''). Subscript indices show which members of an equation are referred to; e. g., (2_3) denotes the third member of (2), and (2_{135}) denotes the equality $(2_1) = (2_3) = (2_5)$.

* For further and instructive examples, see COMBESQUE's French translation of the admirable "Theory of Determinants and their principal Applications," by Prof. BRIOSCHI, of Pavia, 1856, 8vo, pp. 216; for the best elementary exposition of Invariants, &c., see SALMON's Higher Algebra, Dublin, 1859, 8vo, pp. 147; for a condensed and able account of Determinants as applied to Differential Equations, see PEIRCE's Analytic Mechanics, 1855, Chapter X.; † and for the "General Theory of Elimination," see FAA DE BRUNO's work, Paris, 1859, pp. 224; SPOTTISWOODE's *Elementary Theorems relating to Determinants*, 1851, 4to, pp. 63.

† Almost the only other American notice of Determinants which I have seen, is an ingenious and suggestive article "On the Resolution of Symmetrical Equations," by Mr. STOCKWELL, *Astronomical Journal*, VI., 146, 1860.

I have given much thought to the determinant-notation, and especially to that of Chapter III. I have sought to select it from existing systems; but, as the best of these do not perfectly harmonize, I have ventured upon some changes, which, if found conducive to symmetry and convenience, will, I hope, be adopted by others.

Notation should be clearly separated into “temporary,” adopted but for a single essay or part of one, and requiring redefinition if ever used again; and “permanent,” to be used in the same sense in all memoirs by all writers who adopt it, even without redefinition,—yet always changeable *temporarily and by express definition*. But one notation may be permanently used in different senses, if they differ so *widely* or pertain to such different *subjects* as not to interfere. Of course, I have been most careful with such *permanent* notation as I *have not taken from others*.

Further explanations may be given in text or foot-notes where they are needed. Let us now commence our subject again.

[To be Continued.]

Mathematical Monthly Notices.

Concise Mathematical Operations; being a Sequel to the Author's Class-Books. With much additional Matter. By H. N. ROBINSON, LL.D., formerly Professor of Mathematics in the United States Navy, &c. New York: Ivison and Phinney. 1854.

Elements of Analytical Geometry, and the Differential and Integral Calculus. By H. N. ROBINSON, LL.D., Author of a Course of Mathematics, &c. New York: Ivison and Phinney, 48 and 50 Walker Street.

We include these two volumes in the same notice, because in the former, the author first presented his peculiar views of the Differential Calculus, about which we are at present mainly concerned. The latter work, as we learn from the publisher's advertisement, is now undergoing revision, and the two subjects which it contains will be published in separate volumes in January, 1861. As this will give the author a good opportunity to make such changes and improvements as the use of the work may have suggested, we beg to call his attention to a few features of the present edition, feeling desirous of doing all in our power to improve the text-

books from which our students get their first ideas of the science. In the Preface to the work on Mathematical Operations, the author says:—

"There has been a great deal of unnecessary controversy about the Differential and Integral Calculus, which we think can and ought to be wiped away. And we have here given a little foretaste of what we shall attempt if circumstances prompt us to write a work on that subject.

"It is not for us to assume that we can make science clearer than others, but we have yet to see the works of an author who has made the least attempt to show the simple elementary nature of this science. They at once commence with the definition of constants and variables, and then direct what to do.

"We have yet to see the first book that expends a word in giving an idea of what the Calculus is, or what is the utility and object of the science, and we charge more than half the obscurity to this fact alone: hence we could not forbear being a little elementary when we came to that subject, and we leave it to those readers, who have formerly studied other works on this science, to say whether we have or can dispel any of the obscurity that has so long hovered around it."

Again, on page 259:—

"One reason for the appearance of this work is that it is required, because able mathematicians have written so obscurely. They seem to have written as I should, were I indifferent whether the reader, or rather the learner, understood me or not."

We propose now to allow the author to state his own case; and then we can judge whether the "unnecessary controversy" has been "wiped away," either in the "little foretaste" where "we could not forbear being a little elementary," or in the work, which "circumstances" prompted.

The assurance with which the subject is approached, and the estimate which is put upon the labors of those who founded the science, would naturally lead the reader of the above passages to anticipate either great success or great failure, according to his knowledge of the subject and his acquaintance with the labors of the great masters of the science. We shall quote the exact language, italics, and punctuation of the author; and first from the former work:—

"The *differential calculus* is the science of minute variations, or of corresponding *small differences*—a science which owes its birth to the varying elements of Astronomy."—p. 302.

One of the author's correspondents asked him why he called a certain explanation an application of the differential calculus, to which he replied as follows:—

"If the operation I sent you is not calculation, I know not what it is—it may therefore be called calculus; and if in any operation small quantities may be omitted on account of their insignificance in relation to larger quantities, the small difference so omitted constitutes the *differential calculus*, and to obtain that general expression, you will see, by looking on page 193 of the Astronomy, that the powers of r above the first were omitted."—p. 314.

Arithmetic may therefore be called calculus; and because we omit decimal quantities, which on account of their smallness do not affect the required accuracy of the result, it is the differential calculus.

"The differential calculus is a branch of Analytical Geometry. It is a science for computing the *ratio* of small differences."—p. 315.

After giving the differentials of the circular functions, the author says:—

"All can understand how these equations are obtained; but what if we can? says the inquiring student. What use are they? What do we learn by them? It is useless to answer these questions by words only, we must show the answer by the following examples. Because the differential of a logarithm is the differential of the quantity divided by the quantity, therefore $d \log \sin x = \cot x dx$. For example, if we assume $x = 25^\circ$, and also assume $dx = 1'$, the differential, or difference between the $\log \sin 25^\circ$ and the $\log \sin 25^\circ 1'$ is expressed by $m \cot 25^\circ \times 1'$. We might assume $dx = 2'$ as well as $1'$, without error as far as six places of decimals; but it would not do to assume $dx =$ any large number of minutes; hence the differential calculus must be applied with judgment. Those who are naturally more nice than wise are commonly prejudiced against this science, and such frequently say it is no science at all; however, their objections are of no consequence."—pp. 322, 323.

Is it possible that the author does not know the use of TAYLOR's Theorem?

We shall now give a few extracts from the *Elements of the Calculus*. As a first example the

author takes the equation $y = ax + b$, and, calling h the increment of x , finds $y' - y = ah$. He then says:—

"The first member of this equation is obviously the *increment* of y , whatever be the value of h , and when h is *extremely* small in relation to x , (we will not say *infinitely* small, as that word puzzles, and it is unnecessary) then $y' - y$ is extremely small in relation to y and in that case we write dy in place of $y' - y$, and dx in the place of h ."—p. 146.

Here, then, we have the author's theory of the Calculus. About how small he supposes a differential to be we are not informed; at any rate it is unnecessary, it seems, so far as the logic of the subject is concerned, to consider them infinitesimals. Perhaps we are expected to use our judgment here as well as in the applications.

"The terms *fluxion*, *differential*, and *derivative*, all mean the same thing. There are cases, as in *derived polynomials* in algebra, that it would not do to call the derivative a differential, as the increment might be too large."—p. 146.

We might suppose, were it not for the unfortunate reference to derived polynomials in algebra, that this supposed equality of meaning assigned to the derivative was a slip of the pen. But, at any rate, we learn from this quotation that an increment may be "too large."

On pages 149 and 150 we find a reference to the *limiting ratio*, but then it is said to be between the function and its variable. But this simple reference to another theory seems little more than accidental; for when we come to the differentiation of fractions on page 155, the author has got back upon the doctrine of "extremely small (not to say infinitely small)."

Such is our author's theory of the differential calculus; which seems to us strikingly like the calculus of finite differences used as the differential calculus. Indeed, if we were asked, as we have been, to give a definition of our author's differential calculus, we should say that it is a method of finding a change in a function corresponding to a given change in the variable, provided the change in the variable be not too great; in other words, it is a rather coarse way of computing changes in the values of functions using only first differences, and the judgment as to how large we may assume those differences to be.

Next, let us examine some specimens of demonstration, in the light of the theory:—

"Sometimes we have $u = f(y)$, and $y = F(x)$, and require the differential coefficient between the function u and the variable x . For that purpose we first find $\frac{du}{dy}$ from one equation, and then $\frac{dy}{dx}$ from the other, and multiply them together, and we have $\frac{du}{dx}$ as required."—p. 154.

It seems, then, that $\frac{du}{dy}$ and $\frac{dy}{dx}$ are the same as two ordinary fractions, and their product must be reduced to its lowest terms to obtain the required result.

How it follows, that in general, dy , as an arbitrary increment, equals dy as a dependent increment, and especially so, if neither is infinitely small, we are not informed. Indeed it is not a self-evident proposition applicable to functions involving any number of variables, in any theory of the calculus; and the difficulty is simply disguised for the sake of an apparent simplicity, which will deceive none but very young students.

"Hence we observe that whatever value is given to the variable y , *Taylor's theorem*, and the *binomial theorem*, will give the same result. Hence, when one of these theorems *fail* the other *fails*, but we never apply the word *fail* to the binomial theorem, and it is not clear to us that such an expression should ever be applied to Taylor's theorem."

If it be not clear to the author, how will it be to the student? The remark that "the *failure* is in the hypothesis and not in the theorems," shows that the author was probably correct when he said, "it is not clear to us."

The true question is, Does TAYLOR'S Theorem, for any function whatever, fail to give the true development to any number of terms for a given hypothesis, when some other method would not fail?

On pages 199 – 202 the tangent, sub-tangent, &c. of plane curves are found by means of the Differential Calculus, and on page 202 we find the remark that, “the student will perceive that these results are here obtained far more easily than in analytical geometry, but we are indebted to analytical geometry for the primary equation of the curve.” But a note on the bottom of the same page says that, “It is important that the student should observe that this portion of the calculus is pure analytical geometry.”

The idea that the calculus is a branch of analytical geometry is also impressed upon the student on pages 209, 217, 220, 229, as well as in the Preface, apparently, without the faintest suspicion that, as a calculus of functions, with its fundamental principles and its logic, it is entirely independent of any of its applications.

“The differential of an increasing quantity is *positive* before arriving at the maximum zero, and *negative* afterwards as we perceive by merely inspecting the figure, and *this is a general principle*. In like manner the differential of a decreasing quantity is *minus* before it attains its minimum point, it is zero at that point, and *positive* after passing that point. Hence, if the second differential of a function is *minus*, it indicates that the first differential corresponds to a *maximum*, and if *plus* it indicates a *minimum*. The foregoing illustrations are too plain and practical to meet the entire approbation of some minds; therefore, we give the following as more general and abstract.”

Why “some minds” should object to correct geometrical illustrations, and what kind of minds they must be if they do, we cannot understand; but we can comprehend why the “more general and abstract” discussion should not meet the entire approbation of some minds; for a cruder one we never examined. Some of the important points are entirely omitted; such, for instance, as the case in which the same values of the variable reduce the second derivative, as well as the first, to zero. Nothing is said of the maxima and minima of functions of more than one variable.

But we must, at present, omit many other points in the volumes before us for want of space. We have given them, and especially the parts devoted to the Differential and Integral Calculus, a careful and thorough examination; and we are constrained to say that we have never had the misfortune to examine two works, so totally unfitted for text-books, both as regards the treatment of the subjects and the style in which they are written.

We shall take an early opportunity of calling attention to other works by the same author.

Objects and Plan of an Institute of Technology; including, I. A Society of Arts; II. A Museum of Arts; and III. A School of Industrial Science. Proposed to be established in Boston. Prepared by direction of the Committee of Associated Institutions of Science and Arts; and addressed to Manufacturers, Merchants, Mechanics, Agriculturists, and other friends of enlightened industry in the Commonwealth.

WILLIAM B. ROGERS, *Chairman*.

MARSHALL P. WILDER,	JAMES M. BEEBE,	B. S. ROTCH,
SAMUEL H. GOOKIN,	DR. S. CABOT, Jr.,	J. D. PHILBRICK,
ALFRED ORDWAY,	G. W. PRATT,	GEORGE B. EMERSON,
M. D. ROSS,	AMOS BINNEY,	R. C. WATERSTON,
ALEX. H. RICE,	DR. S. KNEELAND, Jr.,	ERASTUS B. BIGELOW,
E. S. TOBEY,	CHARLES L. FLINT,	CHARLES H. DALTON,

Committee.

The Plan contemplates the following Standing Committees. *First*. Committees of Arts. On Mineral Materials; on Organic Materials; on Tools and Instruments; on Machinery and Motive Powers; on Textile Manufactures; on Manufactures of Wood, Leather, Paper, India Rubber, &c.; on Manufactures of Pottery, Glass, and Precious Metals; on Chemical Products and Processes; on Household Economy, including Warming, Illumination, Water-Supply, Ven-

tilation, and the Preparation and Preservation of Food ; on Engineering and Architecture ; on Commerce, Navigation, and Inland Transport ; on the Graphic and Fine Arts. *Second.* Committee on the Museum. *Third.* Committee on the School of Industrial Science. *Fourth.* Committee on Publications. A very well defined outline of the duties of each committee is given ; subject to such changes as experience shall hereafter suggest.

The School of Industrial Science and Art, is to be arranged under the following departments : 1st. A School of Design ; 2d. A School of Mathematics ; 3d. A School of Physics ; 4th. A School of Chemistry ; 5th. A School of Geology. Such is the general outline of a most admirable plan, with the details so arranged as to form a symmetrical and systematic whole.

The Report begins by setting forth in a clear and forcible manner the need now felt for the establishment of such an Institution, and the practical value of the results which would flow from it. In the following opinion we heartily concur : " In New England, and especially in our own Commonwealth, the time has arrived when, as we believe, the interests of Commerce and the Arts, as well as of General Education, call for the most earnest co-operation of intelligent culture with industrial pursuits. Our success hitherto in the competitions of trade, manufactures, and the other productive Arts, has been the admitted result of the superior intelligence which has inspired our enterprise and guided our activity ; but, to secure a steady prosperity in the midst of the busy inventions and rapidly expanding knowledge which mark these pursuits in the leading European nations, we feel that it has become indispensable for us to provide, at least as effectually as they have done, such facilities for practical knowledge, and for the intelligent guidance of enterprise and labor, as may make our progress commensurate, step by step, with the advances of scientific and practical discovery."

We regret that space forbids our giving a full statement of the arrangement of the proposed Museum, and School of Industrial Science and Art. But the whole plan is well matured, and eminently practical ; and of the desirableness of establishing such an Institution in Boston, there can be no question. Still, admitting all that its most enthusiastic friends claim for it, can the idea be carried into execution ? We believe it can and will be done. It will need wise counsellors, and unyielding friends, who will be satisfied with nothing less than the largest measure of success. But these it has the good fortune to possess in the persons of the Committee under whose direction this comprehensive Report has been prepared. It will also need the most ample means ; but these, we are assured by those who know, and have a right to speak in its behalf, will not be wanting ; and it only remains to be seen whether our State will add this last and crowning excellence to her unsurpassed educational system by granting the desired location, upon which private munificence will rear the Institution.

On a Chart and Diagram for Facilitating Great-Circle Sailing. By HUGH GODFRAY, M. A., St. John's College, Cambridge. (From the Transactions of the Cambridge Philosophical Society, Vol. X. Part II.)

On charts, constructed on the *Central or Gnomonic Projection*, great circles are represented by straight lines ; and thus, in Great-Circle Sailing, such charts answer the same purpose that Mercator's Charts do in Rhumb Sailing. To see the ship's track it is only necessary to draw a straight line from the ship's place to her destination. The author says : " By the addition of a Course and Distance Diagram I have made this projection answer all the conditions of Great-Circle Sailing with as much, if not more, facility than Mercator's chart does for sailing on a Rhumb. The track is *seen* a straight line, and this being drawn, the various courses and the distances to be run upon each are obtained, as also the distance from the ship to her destination, by a mere inspection of the diagram ; and the chart can be used like an ordinary one for pricking off the ship's place from day to day."

Accompanying the Chart and Course and Distance Diagram, we have received from the

author a *Time Azimuth Diagram*, with a short memoir in explanation of its object, use, and construction. By means of the Diagram the true azimuth can be obtained without calculation to within one eighth of a degree. The data required are the latitude, sun's declination and apparent time at ship. It is hardly possible that such simple and elegant graphic solutions of these problems should not soon find their way into general use.

On the Secular Variations and Mutual Relations of the Orbits of the Asteroids. By SIMON NEWCOMB, S. B., A.A.S. (From the Memoirs of the American Academy, New Series, Vol. VIII.) Cambridge: Welch, Bigelow, and Co., Printers to the University. 1860.

The particular object of this Memoir is to submit OLBER's hypothesis of the origin of the asteroids to as rigorous a test as is consistent with the nature of the problem. If this hypothesis be true, then there must have been, and still be, certain relations among the asteroid orbits; and the test consists in determining whether these relations do in fact exist. The author has arranged the discussion under the following heads:—

§ 1. *Computation of the rigorous expressions in terms of the times of the elements of the asteroids.*

This section is treated with great elegance and skill. All the terms in these expressions, depending upon the elements of the asteroids, are given in tables with the mean distance of the asteroid as an argument; so that other asteroids may, with but trifling labor, be added to the list of twenty-five treated in this memoir, as fast as their elements become sufficiently well known.

§ 2. *Of the possibility that the orbits of all the asteroids once intersected in a common point.*

Carrying back the elements of the asteroid orbits by means of the formulas in § 1, there is found no tendency to a common point of intersection of the orbits.

§ 3. *Have the elements of the asteroid orbits ever been materially affected by a resisting medium?*
Not sensibly.

§ 4. *Of the relations among the mean distances, eccentricities, and inclinations of the orbits of the asteroids; and between their masses and the velocities with which they must have been projected, if OLBER's hypothesis be true.*

The actual relations are not found to be those which would exist if OLBER's hypothesis be true.

§ 5. *Of certain observed relations among the asteroids which are the necessary or probable result of known causes, and therefore throw no light on the origin of the asteroids.*

This last section is very interesting, and contains satisfactory solutions of several important questions; such as the unequal distribution of the nodes and perihelia of the orbits; the interlinking of certain orbits, &c.

This valuable memoir is by far the most successful discussion ever made to settle one of the most deeply interesting problems in astronomy, which has agitated the scientific world for the last half-century; and will reflect great credit upon the ability of its author.

Plane Trigonometry, crown 8vo, pp. 271; *Spherical Trigonometry*, crown 8vo, pp. 112. For the use of Colleges and Schools. With numerous examples. By I. TODDHUNTER, M. A., Fellow and Assistant Tutor of St. John's College, Cambridge. Macmillan and Co.: Cambridge, and 23 Henrietta Street, Covent Garden, London. 1859.

We do not call the attention of teachers and students to these volumes because we think that there are no good text-books upon these subjects in this country. On the contrary, we have a few excellent ones; and we have no hesitation in saying that we know of no foreign work which surpasses PROFESSOR CHAUVENET's *Treatise on Plane and Spherical Trigonometry* in all the elements of a good text-book, except, perhaps, in its collection of examples for practice. In this respect these volumes are especially worthy the attention of teachers; the one on Plane Trigonometry contains over six hundred examples, and the other about one hundred and fifty. But they are not ordinary works in other respects. We are particularly interested in the

chapters on the Construction of Trigonometrical Tables; Theory of Proportional Parts; Expansions of Trigonometrical Functions; Summation of Trigonometrical Series; Resolution of Trigonometrical Expressions into Factors; Area of a Spherical Triangle. Spherical Excess; Geodetical Operations.

The author, after showing the connection of the values of the old and new Trigonometric Functions, remarks that, "The old definitions give some indications of the origin of the terms *sine*, *cosine*, &c." The word *sine* seems derived from the Latin *sinus*, a bosom, the arc is supposed to represent a bow, and thus gets its name, and the string, half of which represents the sine of half the arc, would come against the breast of the archer. The words *tangent* and *secant* are naturally derived from the old definitions. (See *Penny Cyclopædia*, Art. Trig.)

The modern method has now completely superseded the ancient method in English works; it was introduced by DR. PEACOCK. (See PEACOCK'S *Algebra*, Vol. II. p. 151.) It may, however, be observed, that it is stated by PROFESSOR DE MORGAN (*Trig. and Double Algebra*, p. 18), that RHETICUS, who gave the first complete trigonometrical table, and invented the secant and cosecant to complete it, used the method of ratios."

We find in the *Spherical Trigonometry*, p. 38, the method of proof of NAPIER'S Rules indicated by NAPIER in his *Mirifici Logarithmorum Canonis Descriptio*, pp. 32, 35.

In this connection the author gives a collection of opinions with respect to the practical utility of NAPIER'S Rules. "In the whole compass of mathematical science there cannot be found, perhaps, rules which more completely attain that which is the proper object of rules, namely, facility and brevity of computation." WOODHOUSE'S *Trig.*, Chap. X. "In the opinion of DELAMBRE (and no one was better qualified by experience to give an opinion) these theorems are best recollected by the practical calculator in their unconnected form." (AIRY'S *Trig.*, *Encyclopædia Metropolitana*.) "There are certain mnemonical formulæ called NAPIER'S *Rules of Circular Parts*, which are generally explained. We do not give them, because we are convinced that they only create confusion instead of assisting the memory."

We had always supposed by dividing a circle into five parts, and writing upon them in order, $a, \frac{\pi}{2} - B, \frac{\pi}{2} - c, \frac{\pi}{2} - A, b$; then selecting any one of these five quantities, and observing that of the remaining four, two ($b, \frac{\pi}{2} - B$) are adjacent and two ($\frac{\pi}{2} - c, \frac{\pi}{2} - A$) opposite; that the rules

$$\begin{aligned} \sin a &= \tan b \tan \left(\frac{\pi}{2} - B \right) = \cos \left(\frac{\pi}{2} - c \right) \cos \left(\frac{\pi}{2} - A \right) \\ &= \tan b \cot B \qquad \qquad = \sin c \sin A, \end{aligned}$$

in which the mind associates *sine* and *middle*, *tangent* and *adjacent*, *cosine* and *opposite*, could not be forgotten or misapplied by the student. Such we should think, from the admirable and lucid manner in which the subject is presented, must be the author's opinion.

Our examination of these volumes confirms us in the high opinion we entertain of the author's series of text-books.

Modern Text-Books in the Sciences.

A. S. BARNES AND BURR,
PUBLISHERS.

Chemistry, Geology, and Botany.

Porter's First Book of Chemistry and Allied Sciences.

Including an Outline of Agricultural Chemistry. By JOHN A. PORTER, A. M., Professor of Agriculture and Organic Chemistry at Yale College. 12mo. Cloth. Price, 50 cents.

Porter's Principles of Chemistry.

Embracing the most Recent Discoveries in the Science, and the Outlines of its Application to Agriculture and the Arts. Illustrated by numerous Experiments. By JOHN A. PORTER. 12mo. Cloth. Price, \$1.00.

This work has been prepared expressly for Public and Union Schools, Academies, and Seminaries, where an extensive course of study on this subject and expensive apparatus was not desired, or could not be afforded. A fair, practical knowledge of Chemistry is exceedingly desirable and almost a necessity at the present day, but it has been taught at very few Public or Union Schools, owing entirely to the want of suitable text-books, adapted to simple apparatus, or such as could be readily obtained. It is confidently believed that this work supplies this great want, and will be found in every respect just what is required.

Norton on Porter's First Book of Science.

Designed as an Introduction to the Study of Sciences. — PART I. Embraces Natural Philosophy and Astronomy, by WILLIAM A. NORTON, of Yale College. PART II. Chemistry, Physiology, Geology, and Agricultural Chemistry. Price, \$1.00.

Gregory's Hand-Book of Organic Chemistry.

For the use of Students. By WILLIAM GREGORY, Professor of Chemistry in the University of Edinburgh. Edited by J. M. SANDERS. 480 pp., 8vo. Cloth. Price, \$1.50.

Gregory's Hand-Book of Inorganic Chemistry.

For the use of Students. By WILLIAM GREGORY, Professor of Chemistry in the University of Edinburgh. Edited by J. M. SANDERS. 426 pp., 8vo. Cloth. Price, \$1.50.

These works are intended for the use of advanced students in this interesting science. They are the text-books in some of our leading colleges, and are well recommended by those who have used them.

Page's Elements of Geology.

By DAVID PAGE. Illustrated. 392 pp., 12mo. Cloth. Price, 75 cents.

The subject is here presented in its two aspects of interesting and important. Interesting, inasmuch as it exhibits the progressive conditions of the earth from the remotest periods, and reveals the character of the plants and animals which have successively adorned and peopled its surface; and important, as it determines the position of those metals and minerals upon which the arts and manufactures so intimately depend.

Emmons's Manual of Geology.

Designed for the use of Colleges and Academies. By EBENEZER EMMONS, State Geologist of North Carolina, late State Geologist of New York, Professor of Natural History and Geology in Williams College, etc. Illustrated with numerous engravings. 297 pp., 8vo. Cloth. Price, \$1.50.

A work that really requires nothing more than the name and reputation of the author to recommend it. Its intrinsic merit is such as must entitle it to a prominent position among kindred text-books.

Wood's Class-Book of Botany.

A new edition. Price, \$2.00.

Prof. Wood is now engaged in rewriting his celebrated Class-Book of Botany, embracing all the recent improvements, and will be unequalled by any now published. This splendid work will be ready January 1st, 1860.

Botany of the Southern States.

In Two Parts. PART I. Structural and Physiological Botany and Vegetable Products. PART II. Descriptions of Southern Plants. Arranged on the Natural System. Preceded by a Linnean and a Dichotomous Analysis. By JOHN DARBY, A. M., Professor of Natural Sciences in Auburn College, Ala. 612 pp., 12mo. Cloth. Price, \$1.00.

This work constitutes an outline of Botany in all its most important relations as a science. It has been the labor of the author for many years to bring it as near perfection as possible. It may be asserted that no work before the public affords so easy a means for the analysis of plants as this.

PUBLISHED BY A. S. BARNES & BURR,
51 and 53 John Street, New York.

Publishers of the National Series of Standard School-Books.

Modern Text-Books in the Sciences.

A. S. BARNES AND BURR,
PUBLISHERS.

Teachers and Scientific Men will please examine the following Text-Books, designed for Schools, Academies, and Colleges.

Single copies sent for examination AT ONE HALF THE RETAIL PRICE.

NATURAL PHILOSOPHY.

1. Norton's First Book of Natural Philosophy and Astronomy.

By WILLIAM A. NORTON, Professor of Civil Engineering in Yale College. 12mo. Cloth. Price, 50 cents.

The present little volume treats of the Elements of Natural Science, and is designed to meet the wants of young persons who do not intend to pursue a complete course of academical study. It is a good introduction to a more complete manual.

2. Ganot's Popular Physics.

By W. G. PECK. Introductory Course of Natural Philosophy, for the Use of Schools and Academies. Edited from "Ganot's Popular Physics." Price, \$1.00.

This work is illustrated by over 300 splendid engravings, and pronounced by all who have seen it as far superior to anything of the kind before the public. 10,000 copies have already been printed.

3. Peck's Elements of Mechanics.

Designed for Colleges, Academies, and High Schools. By WILLIAM G. PECK, M. A., Professor of Mathematics in Columbia College. 12mo. Cloth. Price, \$1.25.

A need has long been felt, by those who have classes in this science, of a text-book which may occupy an intermediate ground between those higher treatises which are found too difficult to be read with profit, save by a few, and too extensive to be studied, even by the few, in the limited time allotted to this branch of education, and those more elementary works, too simple for advanced classes, and, because of their non-mathematical character, not adapted to prepare the student for subsequent investigations in science. Professor Peck's work, just from the press, has been prepared with an especial view to meet this want. It will be found to contain a sufficient amount of information for those who lack either the leisure or desire to make mathematical sciences a specialty; and it will serve as a suitable introduction to those higher treatises on Mechanical Philosophy, which all must study who would appreciate and keep pace with the wonderful discoveries that are daily made in science.

4. Norton and Porter's First Book of Science.

Designed for Public and Private Schools. By W. A. NORTON and J. A. PORTER, Profes-

sors of Yale College. In two parts. Part I. embraces Natural Philosophy and Astronomy; Part II. Chemistry and Allied Sciences. 12mo. Cloth. Price of each part, separate, 50 cents; or two parts in one, \$1.00.

This work forms the most complete and interesting compendium of Familiar Science for young children now published. It embraces the principles of Natural Philosophy, Astronomy, Chemistry, Physiology, and Geology. The catechetical plan, found the most effective in impressing the minds of the younger class of pupils, has been adopted. Each subject is divested of the *dryness of detail* which is apt to characterize books of this grade, and is treated in a manner fitted to prepare the mind for future investigation, without wearying with present superfluity. It is thought to be eminently adapted to impart a practical knowledge of familiar things, and enable a child to understand the *why* and *wherefore* of the various facts and phenomena of every-day life.

5. A New Treatise on Astronomy and the Use of the Globes.

Containing Definitions, Motions, and Positions of the Sun, Moon, and Planets; Kepler's Laws, and the Theory of Gravitation; Gravitation; Refraction; Twilight and Parallax; Connections, Periods, Distances, Phenomena, and Magnitudes of the Heavenly Bodies composing the Solar System, &c.; also, an extensive collection of the most useful problems on the Globes, illustrated by a suitable variety of examples. By JAMES MCINTYRE, M. D., Professor of Mathematics and Astronomy in the Central High School of Baltimore. 12mo. Cloth. Price, 75 cents.

6. Natural Philosophy for Colleges.

By W. H. C. BARTLETT, LL. D., Professor of Natural Philosophy in the Military Academy of the United States.

BARTLETT'S SYNTHETIC MECHANICS.....\$3.00
BARTLETT'S ACOUSTICS AND OPTICS.....2.00
BARTLETT'S SPHERICAL ASTRONOMY.....2.50
BARTLETT'S ANALYTICAL MECHANICS.....3.50

7. Mrs. Willard's Astronomical Geography.

With the Use of the Globes. Arranged either for simultaneous Reading and Study in Classes, or for study in the common method. By EMMA WILLARD. Price, 75 cents.

PUBLISHED BY A. S. BARNES & BURR,
51 and 53 John Street, New York.

Publishers of the National Series of Standard School-Books.

December,

Worcester's Quarto Dictionary The Standard

VERY SIGNIFICANT FACTS.

The following recommendations are from some of the most distinguished American and English scholars. They are but a few from many which have been received, testifying to the superiority of Worcester's Quarto Dictionary. These testimonials are of the highest value, for they have all been given during the present year, and after an examination of this work and of that which is endeavoring to hold the position of a rival. The scholars of America and of England, with scarcely an exception, have decided in favor of Worcester. Not a single scholar, equal in authority to any one mentioned below, can be cited as giving, after a comparison of the two works, the preference to Webster's Dictionary. We give the testimony:—

From C. C. FELTON, LL.D., *President of Harvard College.*

Aware of the labor and care which had been devoted to this (the department of scientific terms) as well as to other parts of the work, I felt assured that Worcester's Quarto Dictionary would more nearly meet the public wants than any other hitherto published.

My expectations have been more than fulfilled. I find it not only rich beyond example in its vocabulary, but carefully elaborate in all the details, and thoroughly trustworthy as a guide to the most correct and elegant usage of the language.

From the REV. JOSEPH BOSWORTH, D.D., F.R.S., *Professor of Anglo-Saxon, Oxford, England.*

It is the most complete and practical, the very best, as well as the cheapest English Dictionary that I know.

From GEORGE P. MARSH, LL.D.

The work of Dr. Worcester is unquestionably much superior to any other general dictionary of the language in every one of these particulars (orthography, pronunciation, definition, fulness of vocabulary, and precision and distinctness of definition).

From REV. W. WHEWELL, D.D., *Master of Trinity College, England.*

I have repeatedly consulted the Dictionary since it has been in my possession, and have seen reason to think it more complete and exact than its predecessors.

From CHARLES RICHARDSON, LL.D., *the oldest living English Lexicographer, England.*

I sincerely hope you may enjoy from your brethren, both in America and England, that tribute of honor to which you have earned so undoubted a title.

From D. R. GOODWIN, D.D., *President of Trinity College, Hartford.*

It was but a short time since that I was led to commend another dictionary as, on the whole, and with some exceptions, the best and most complete thing of the kind within my knowledge. The commendation was honestly given at the time; but now it must be withdrawn in favor of yours. I consider your dictionary, in orthography, pronunciation, and definitions, as superior to any of its predecessors.

From REV. W. B. SPRAGUE, D.D., *of Albany, N. Y.*

My opinion of Worcester's Quarto Dictionary, after having given it as extended an examination as my circumstances would admit, is, that there is no other dictionary in the language that compares with it for completeness, accuracy, comprehensiveness, and precision, and perhaps I ought to add, that I have arrived at this conclusion rather contrary to a preconceived opinion.

From REV. HENRY A. BOARDMAN, D.D., *of Philadelphia.*

I particularly like it (the Dictionary), 1. Because of its very comprehensive character; 2. Because it adheres to the settled orthography of our noble language,—discarding those innovations which, however countenanced by certain publishing-houses, have never to any extent been accepted by the scholars of our country.

From LOUIS AGASSIZ, LL.D.

It is of great importance, when the nomenclature of science is gradually creeping into common use, that an English lexicon should embrace as much of it as is consistent with the language we speak. I am truly surprised and highly delighted to find you have succeeded far beyond my expectations in making the proper selection, and combining with it a remarkable degree of accuracy. More could hardly be given except in a scientific cyclopædia.

The following lines are quoted from Harper's Magazine for September. They serve to show very truthfully the comparative value of recent and old commendations:—

"INJUSTICE.—Our attention has been called to a species of injustice of which publishers are sometimes guilty, in publishing commendations of school-books without giving the dates when they were written. Especially does this merit reproof when these commendations are old, and when it is known that the writers have subsequently commended other and later publications in the same department. It will readily be seen that this is frequently not only an act of injustice to teachers who have had the courtesy to commend a book, but that it is also a fraud upon the public."

SWAN, BREWER, & TILESTON,

131 Washington Street, Boston.

1860.

No. 117 Washington Street, Boston,
October, 1860.

CROSBY, NICHOLS, LEE & CO.

HAVE JUST PUBLISHED

THE WORKS OF CHARLES LAMB. With a fine Portrait.
In Four Volumes. 12mo. Price, \$5.00.

This new edition of Lamb is reprinted from the latest London edition, edited by Talfourd, contains his *Life, Letters, and Final Memorials*, his *Essays of Elia*, *Rosamund Gray*, with other *Essays and Poems*, being the best and most complete edition issued.

It is elegantly printed on paper slightly tinted, and is in every respect an edition worthy of the author.

"This is the most convenient and elegant edition of Lamb's Works which has been published, either in England or in this country, and it leaves nothing to be desired in respect to typographical beauty or any other essential of a good library edition."—*North American Review.*

The Three Great Histories,—Hume, Gibbon, and Macaulay.

Elegant Library Editions, beautifully printed on tinted paper, convenient in size, and withal the best and cheapest yet published.

THE HISTORY OF ENGLAND, FROM THE INVASION OF JULIUS CÆSAR TO THE ABDICATION OF JAMES THE SECOND, 1688. By DAVID HUME, Esq. A new Edition; with the Author's last Corrections and Improvements. To which is prefixed a short Account of his Life, written by himself. In 6 volumes. 12mo. With a Portrait. Price, \$7.50.

SIR JAMES MACINTOSH says of Hume: "His greatest work, and that which claims most attention, was his 'History of England,' which, notwithstanding great defects, will probably be at last placed at the head of historical compositions. No other narrative seems to unite, in the same degree, the two qualities of being instructive and affecting. No historian approached him in the union of the talent of painting pathetic scenes with that of exhibiting comprehensive views of human affairs. . . . His narrative is flowing and various; in common events, short and clear; in great actions, rapid and animated; in affecting incidents, circumstantial and picturesque. His general observations seem always to be required by his subject; the most profound ideas are clothed in a transparent simplicity."

THE HISTORY OF THE DECLINE AND FALL OF THE ROMAN EMPIRE. By EDWARD GIBBON, Esq. With Notes by the Rev. H. H. MILMAN. A new edition. To which is added a complete Index of the whole Work. In 6 volumes. 12mo. With a Portrait. Price, \$7.50.

Of the various editions of GIBBON this of MILMAN is uniformly acknowledged to have no rival.

PROF. SMYTH, in his *Lectures on Modern History*, says: "But the whole is notwithstanding, such an assemblage of merits, so various, so interesting, and so rare, that the History of the Decline and Fall must always be considered as one of the most extraordinary monuments that have appeared of the literary powers of a single mind, and its fame can perish only with the civilization of the world."

THE HISTORY OF ENGLAND FROM THE ACCESSION OF JAMES II. By THOMAS BABINGTON MACAULAY. In 4 volumes. 12mo. Price, \$5.00.

"It is as fluent and as much colored as Livy; as close and coherent as Thucydides; with far more real condensation and a larger thoughtfulness than either; and quite free from the affected laconisms and sarcasms and epigrams of Tacitus. I do not know that I ever read anything so good as the first forty pages; so clear, comprehensive, and concise, so pregnant with deep thought, so suggestive of great views, and grand and memorable distinctions."—*LORD JEFFREY.*

CURIOSITIES OF LITERATURE. By ISAAC DISRAELI. With a View of the Life and Writings of the Author, by his Son. In 4 volumes. 12mo. Price, \$5.00.

"It embraces a more copious selection of noteworthy facts in regard to books and authors than can be found in almost any other compilation of the kind, while it doubtless owes much of its popularity with all classes of readers to the agreeable form in which these facts are presented."—*North American Review*, April, 1860.

THE NORTH AMERICAN REVIEW. No. CLXXXIX.—For October, 1860.

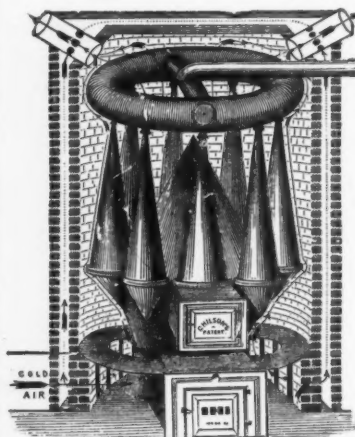
CONTENTS.—Article I. Homer and his Heroines. II. Climatology. III. Life and Labors of Thomas Prince. IV. Edmund Waller. V. Lord Shaftesbury. VI. Second Volume of Palfrey's "History of New England." VII. Quarantine and Hygiene. VIII. Rush's Occasional Productions. IX. The English Language in America. X. The Origin of Species. XI. An "Inglorious Milton." XII. Critical Notices. XIII. New Publications.

This Review has for nearly fifty years stood at the head of the literature of our country, and has numbered among its contributors most of our eminent literary men. The writings of Webster, Everett, Bancroft, and Prescott were first made known through this work; and its pages are now filled by articles from our most prominent writers.

The NORTH AMERICAN REVIEW is published quarterly, in numbers of nearly three hundred pages each, at five dollars a year.

CHILSON'S PATENT CONE FURNACE.

PATENTED IN
AMERICA, ENGLAND, AND FRANCE.



The subscriber begs to call the attention of scientific gentlemen and others to his discovery of a principle for burning fuel, by which all the combustible properties of the smoke and gases eliminated from the burning fuel are thoroughly consumed and their heat retained and made available to the fullest extent, so that there shall be no waste whatever. This important object is accomplished by the use of a large cluster of cones or tapering radiators placed over a broad, shallow, flaring fire-pot, and surmounted by a radiating and exhaustive annular chamber, as represented in the cut. The cones being large at their base and small

at the tops, causes the smoke and gases to be suspended directly over and near the fire, where they are thoroughly consumed, and their heat forced to impinge against and up the cones, thereby creating a powerful radiation.

This plan of furnace secures a fresh, healthful heat, free from that scorching, dry, disagreeable, red-hot iron heat and coal-gas so common to hot-furnaces generally. Persons interested in these matters will here find an invention worthy of their attention, which saves from one half to two thirds of the fuel usually consumed.

COOKING RANGES.

CHILSON'S PATENT DOUBLE OVEN RANGE.

The most important Improvements yet made in Ranges.

The highly satisfactory testimony received from the large number of these Ranges in use in the past three years should certainly induce those in want to examine this Range, which is entirely new in principle, and different from any other before known. It has been got up very perfect and substantial, so that it commends itself at once to the purchaser, in preference to anything else of the Range kind in this country.

Its great economy in the use of fuel, its quick and even baking ovens, its nice arrangement of flues, — without the usual vexatious complication of dampers, — its cast-iron flue plates, preventing any loss of heat in the brick-work, its nice arrangement for roasting meats in the ventilated oven, &c., will be appreciated by every housekeeper. Sizes in variety for dwellings, hotels, &c., with or without Water Backs for heating water for bathing, &c., Hot Closets, and Hot Air Fixtures for heating additional rooms.

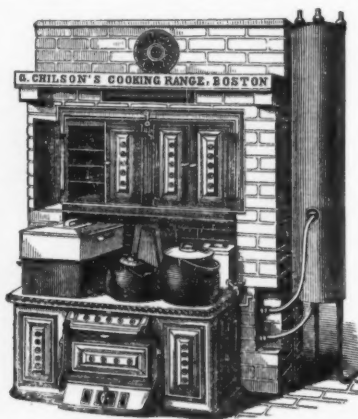
Also, a full assortment of

Chilson's Patent Trio Portable Furnaces, Trio Stoves, Parlor Stoves, Registers, Ventilators, &c., &c.

WAREROOMS 99 & 101 BLACKSTONE ST., BOSTON.

FOUNDRY AT MANSFIELD, MASS.

GARDNER CHILSON.



Philosophical Instruments AND SCHOOL APPARATUS,

MANUFACTURED BY

E. S. RITCHIE, BOSTON, MASS.

Few branches of study in our schools are of more practical importance than Natural Philosophy, and it is only by the use of proper apparatus that its principles can be adequately elucidated.

Until a comparatively recent period, the attention bestowed upon this subject has been confined to a few of the elementary principles; of late years, however, the growing interest in physical science, as a branch of common education, has created a desire for more ample and varied means of experimental demonstration.

It has been my earnest desire and effort to keep pace with the progress of natural science, and, as new principles have been discovered or developed, to devise and construct new instruments, to improve the old, and to add to my list many which heretofore could only be procured from Europe; and also that, in superior quality and finish, as well as in graceful form and proportions, whatever goes from my manufactory shall take rank with the best products of European make.

So numerous and important are the changes that have consequently been made, that I have published a new and enlarged **Catalogue of 100 pages octavo**, containing descriptions of over

Twelve Hundred Pieces of Apparatus,

and illustrated by over **Three Hundred** finely executed engravings, and also many commendatory letters from the most distinguished Physicists in the country.

E. S. RITCHIE.

Philosophical Rooms, No. 313 Washington Street, Boston.

Catalogues will be sent, prepaid, by mail to order, — please enclose 25 cents in stamps.

PHOTOGRAPHING ON WOOD.

BRICHER AND RUSSELL,

Photographers, Designers, and Engravers
ON WOOD.

129 WASHINGTON STREET, BOSTON.

BRICHER AND RUSSELL, by their new process of Photographing on Wood, are now enabled to produce much more accurate representations of objects than by the old and slow process of hand drawing. They will furnish Portraits, Views of Colleges, Seminaries, Academies, and Public Buildings generally, College and Society Seals, Philosophical Apparatus, Mathematical Instruments, and Diagrams (see specimens in Mathematical Monthly), Illustrations for School Books of all kinds, &c., &c.

Directions.

Get a Photographer to take a negative of the object you wish to have engraved, and of the required size, — that is, send us a photograph of the required size, and the reverse of what is wanted when printed.

If a negative photograph cannot be obtained, get as sharp and clear an ambrotype as possible.

Robinson's Complete Course of Mathematics

FOR

SCHOOLS, ACADEMIES, AND COLLEGES.

This Series of Mathematics contains **EIGHTEEN VOLUMES** of well graded and progressive text-books, carefully and thoroughly prepared, in which the author has been assisted by some of the best practical, mathematical talent in the country. These books are adapted to the wants of every grade of scholarship, from the beginner to the civil engineer, and college graduate.

Robinson's Progressive Primary Arithmetic, . . . \$ 0.15	Robinson's Key to University Algebra, . . . \$ 1.00
Robinson's Progressive Intellectual Arithmetic,25	Robinson's Geometry and Trigonometry, (New) . . . 1.50
Robinson's Progressive Practical Arithmetic,56	Robinson's Surveying and Navigation, 1.50
Robinson's Key to Practical Arithmetic,50	Robinson's Analytical Geometry and Conic Sections, 1.50
Robinson's Progressive Higher Arithmetic,75	Robinson's Differential and Integral Calculus, . . . 1.50
Robinson's Key to Higher Arithmetic,75	Robinson's Elementary Astronomy,75
Robinson's New Elementary Algebra,75	Robinson's University Astronomy, 1.75
Robinson's Key to Elementary Algebra,25	Robinson's Mathematical Operations, 2.25
Robinson's University Algebra, 1.25	Robinson's Key to Algebra, Geometry, etc. . . . 1.50

ROBINSON'S

New Geometry and Trigonometry,

EMBRACING

PLANE AND SOLID GEOMETRY, PLANE AND SPHERICAL
TRIGONOMETRY, WITH THE COMMON LOGARITHMIC
TABLES.


In this new book many important theorems have been added, and the methods of demonstration in many instances have been entirely changed. New and original demonstrations of some of the more important principles have been given, and the practical problems and applications both in the Geometry and the Trigonometry have been greatly increased.


Robinson's

Differential and Integral Calculus,

in a separate volume, *rewritten and enlarged*, will be published in January, 1861.

This work will be fully and clearly illustrated, and numerous applications made of principles to practical problems.

 A NEW DESCRIPTIVE CATALOGUE (of 160 pages), and the "EDUCATIONAL CIRCULAR," with matter interesting to Teachers, descriptions, notices, testimonials, prices, etc., sent free, on request.

 Liberal Terms for Specimen Copies and first supplies for Schools.

IVISON, PHINNEY, & CO.,

Publishers, 48 and 50 Walker Street, New York.

SOLD BY BOOKSELLERS GENERALLY EVERYWHERE.

Robinson's

Analytical Geometry and Conic Sections,

new and enlarged, will be ready in January, 1861.

The author in preparing this work will endeavor to make every part of the subjects clearly comprehensible to common minds, so that every one who has been moderately successful in acquiring a knowledge of Algebra can understand Analytical Geometry.

The Key to Robinson's

Algebra, Geometry, Surveying, Calculus, etc.

in one volume, will be ready in January, 1861.

This will be an invaluable work for both the teacher and the student. It will contain the most concise and *best* solutions of all the problems in Robinson's Higher Mathematics.

The other books of the Series will remain as they now are.

ROSS' AMERICAN SCHOOL FURNITURE WORKS,

Established, 1838.

The Pioneer and most Extensive Establishment in the United States.



The above Engraving represents Ross' Improved High School, Normal School, or Academy Double-Column Desk and Chairs; Falls to lift; plain top.



The above Engraving represents Ross' Improved High School, Normal School, or Academy Single-Column Desk and Chair; Fall to lift; plain top.



The above Engraving represents Ross' Improved High School, Normal School, or Academy Single Desk and Chair; Fall to lift; plain top.



The above Engraving represents Ross' Improved High School, Normal School, or Academy Double Desk and Chairs; Falls to lift; plain top.

An Illustrated Catalogue and Information forwarded, on Application, by Mail or Otherwise.

N. B. *Every Article of School Furniture from this Establishment WILL BE WARRANTED. All Communications may be addressed to*

JOSEPH L. ROSS,
Office--**CHARDON, opp. HAWKINS STREET,**
(Near the Revere House,) **BOSTON, Mass.**

Entered according to Act of Congress, in the Year 1860, by JOSEPH L. ROSS, in the Clerk's Office of the District Court of Massachusetts.

MATHEMATICAL BOOKS,

IMPORTED AND FOR SALE BY

SEVER AND FRANCIS, CAMBRIDGE,

SUCCESSORS TO JOHN BARTLETT.

ENGLISH.

American Ephemeris and Nautical Almanac. For the Years 1855-61.	\$1.50
Each	
BAKER. Treatise on Statics and Dynamics	0.30
BLAND. Algebraical Problems. 8vo. Second-hand	1.50
Key to do. 8vo. do.	1.75
Philosophical Problems. 8vo. do.	1.50
Geometrical Problems. 8vo. do.	1.50
BOOLE. Treatise on Differential Equations	4.20
BOOTH. Theory of Elliptic Integrals. 8vo.	2.25
BORDEN. System of Railway Formule. 8vo.	2.25
BRITISH NAUTICAL ALMANAC. 1851 and 1852, each	1.00
CARMICHAEL. Treatise on the Calculus of Operations. 8vo.	2.75
CHAUVENET. Trigonometry. 8vo.	1.50
COURTENAY. Calculus. 8vo.	2.50
COOMBE. Solutions of the Cambridge Problems, 1840, 1841. 8vo.	2.25
DEMORGAN. Differential and Integral Calculus	2.00
EARNSHAW. Dynamics. 8vo.	4.25
FISCHER. Logarithmic Tables of Seven Places. Translated from Bremiker's Vega. 8vo. Hf. mor.	2.50
FROST. Mathematical Questions of the Senate-House Examination Papers. 1838-1849. 8vo.	3.00
GAUSS. Theoria Motus. Translated by C. H. Davis. 4to.	5.00
GREGORY. Differential and Integral Calculus by Walton. 8vo.	5.50
HADDON. Differential Calculus	3.00
HAMILTON. Lectures on Quaternions. 8vo.	6.30
HAYN. Integral Calculus	3.00
HEATHER. Descriptive Geometry	3.30
HEMMING. Differential and Integral Calculus. 8vo.	2.75
HUTTON. Mathematics	4.00
Recreations. 1 vol. 8vo.	4.00
Mathematical Tables. 8vo.	3.75
HYMER. Plane and Spherical Trigonometry	2.50
Algebraic Equations	3.00
Treatise on Conic Sections. 8vo.	2.75
Differential Equations and Calculus of Finite Differences. 8vo.	3.60
Integral Calculus	3.00
Geometry of Three Dimensions. 8vo.	3.00
JAMIESON. Solutions of the Senate-House Rider. 8vo.	2.25
LARDNER. System of Algebraic Geometry. 8vo. Second-hand	1.75
MAIN and BROWN. Indicator and Dynamometer	1.40
Mathematical Monthly. Edited by J. D. Runkle. Vol. I. In Nos. Do. Hf. mor.	3.00
Mathematical Problems and Examples of the Senate-House Examination Papers. 1821-1835. With an Appendix containing the Senate-House Questions for 1837. 8vo.	3.00
PARKINSON. Elementary Mechanics. Post 8vo.	2.57
PEARCE. Analytic Mechanics. 4to.	7.50
PEARCE, J. M. Analytic Geometry. 8vo.	1.50
PEARCE. Elementary Hydrostatics. Post 8vo.	1.03
PRICE. Differential and Integral Calculus. 3 vols. 8vo.	14.50
Quarterly Journal of Pure and Applied Mathematics. Sylvester, Ferrers, & Co. \$6.00 per annum, post-paid.	
RANKINE. Manual of Applied Mechanics	2.00
SALMON. Conic Sections	3.50
Higher Plane Curves	3.50
Higher Algebra	1.80
Solutions of the Cambridge Problems. 1848-1851. Ferrers and Jackson. 8vo.	4.50
1854. Walton and McKenzie. 8vo.	2.50
1857. Campion and Walton. 8vo.	2.50
Senate-House Riders. Jamieson. 8vo.	3.25
SPOTTISWOODE. Elementary Theorems relating to Determinants. 4to.	1.50
TATE. Exercises on Mechanics, with Key. 2 vols.	1.72
Mechanical Philosophy. 8vo.	3.25
Materials. 8vo.	1.75
TATE and STEEL. Dynamics. 8vo.	3.00
TODHUNTER. Analytical Statics. Post 8vo.	3.00
Differential and Integral Calculus. 2 vols. Post 8vo.	6.00
WALTON. Problems illustrative of Plane Coördinate Geometry. 8vo.	4.80
Mechanical Problems. 8vo.	5.50
Problems in Hydrostatics and Hydrodynamics. 8vo.	3.00
On the Differential Calculus. 8vo.	3.25
WALTON and MCKENZIE. Solutions of the Cambridge Problems. 1854. 8vo. \$3.25. Do. 1857.	2.50
Problems in Elementary Mechanics. 8vo.	3.00
WAUD. Algebraic Geometry. 8vo. Hf. cf. (Second-hand.)	1.50
WHEWELL. Analytic Statics. 8vo.	2.25
WILSON. Dynamics. 8vo.	2.87
WOOLHOUSE. Differential Calculus	3.30

FRENCH, &c.

AUDÉ. Poussée des Terres. 8vo. Hf. cf.	\$1.87
BELANGER. Résumé de Leçons de Géométrie Analytique et de Calcul Infinitésimal. 8vo.	2.75
BIOT. Traité des Equations différentielles. Hf. mor.	2.00
BOURCHARLAT. Eléments de Calcul Differential et du Calcul Integral. 8vo.	8.00
BONNET. Leçons de Mécanique Élémentaire à l'Usage des Candidats: l'Ecole Polytechnique. 8vo.	2.00
BRIOSCHE. Théorie des Determinants. 8vo.	2.00
BOURDON. Application de l'Algèbre à la Géométrie. 8vo. Hf. cf.	3.00
CALLET. Tables de Logarithmes. 8vo. Hf. cf.	5.00
CHASLES. Traité de Géométrie Supérieure. 8vo. Hf. cf.	5.00
CHOQUET. Traité élémentaire d'Algèbre. 8vo. Hf. cf.	2.00
COURNOT. Traité élémentaire de la Théorie des Fonctions, et du Calcul Infinitésimal. 2 vols. 8vo.	4.00
CRELLE. Journal für die reine und angewandte Mathematik. Per annum	4.00
DELISLE. Géométrie Analytique. 8vo. Hf. cf.	3.00
DUHAMEL. Cours d'Analyse Calcul Infinitésimal. 2 vols. 8vo. Hf. mor.	5.00
Cours de Mécanique. 2 vols. 8vo. Hf. mor.	4.50
DUPIN. Application de Géométrie et de Mécanique à la Marine aux Ponts et Chaussées. 4to. Hf. mor.	4.50
Developpements de Géométrie. 4to. Hf. mor.	4.50
Géométrie et Mécanique, appliquées aux Arts. 8vo. Hf. mor.	2.50
FRANCEUR. Cours complet de Mathématiques pures. 2 vols. 8vo.	2.00
Eléments de Statique. 8vo. Hf. cf.	2.00
GAUSS. Méthode des Moindres Carrés. 8vo. Hf. cf.	1.03
KÖHLER. Logarithmisch Trigonometrisches Handbuch. 8vo.	2.12
LACROIX. Traité du Calcul Differential et du Calcul Integral. 3 vols. in 4to, avec 18 pl. Mor.	35.00
LAGRANGE. Mécanique Analytique. 2 vols. 4to.	13.00
Théorie des Fonctions Analytiques. 4to. Hf. mor.	6.00
Traité des Equations Numeriques de tous les Degrés. 4to. Hf. mor.	5.00
LALANDE. Tables de Logarithmes. 18mo.	1.25
LAPLACE. Système du Monde. 4to. Hf. mor.	5.50
LEGENDRE. Traité des Fonctions Elliptiques. 3 vols. 4to. Hf. mor.	20.00
LEROY. Traité de Stéréotomie comprenant les Applications de la Géométrie Descriptive à la Théorie des Ombres, la Perspective Lineaire, la Gnomonique, la Coupe des Pierres, et la Charpente. In 4to, avec Atlas de 74 pl. in fol. Paper, \$9.00; hf. mor.	12.00
Traité de Géométrie Descriptive. In 4to, avec Atlas de 71 pl. Paper, \$3.00; hf. mor. \$5.00 and	6.00
Analyse Appliquée à la Géométrie des trois Dimensions. 8vo. Hf. cf.	2.25
LIIOVILLE. Journal de Mathématiques Pures et Appliquées. 1836-1857. 22 vols.	
MAHISTRE. Cours de Mécanique Appliquée. 8vo.	8.00
MONGE. Application de l'Analyse à la Géométrie. 4to. Hf. mor.	10.50
Traité élémentaire de Statique. 8vo. Hf. mor.	1.75
MORIN. Aide-mémoire de Mécanique Pratique. 8vo.	2.00
MONTFERRIER. Encyclopédie Mathématique ou Exposition complète de Toutes les Branches des Mathématiques d'après les Principes de la Philosophie des Mathématiques de Hoëné Wronski. 3 vols. 8vo. To be continued in monthly Livraisons.	6.00
NAVIER. (Ketten) Hängbrücken. 4to.	5.00
OLIVIER. Théorie Géométrique des Engrenages. 4to. Hf. mor.	3.75
Developpements de Géométrie Descriptive. 2 vols. 4to. Hf. mor.	9.00
Complément de Géométrie Descriptive. 2 vols. 4to.	9.00
Mémoires de Géométrie Descriptive, Théorique, et Appliquée. 2 vols. 4to. Hf. mor.	9.00
Application de la Géométrie Descriptive. 2 vols. 4to.	10.00
Cours de Géométrie Descriptive. 2 vols. 4to. Hf. mor.	10.00
POINSON. Eléments de Statique. 8vo. Hf. cf.	2.50
POISSON. Mécanique. 2 vols. 8vo.	6.00
PONCELET et LESBROS. Experiences Hydrauliques. 4to. Hf. mor.	4.00
PONCECOULANT. Système du Monde. 4 vols. 8vo. Hf. cf.	16.00
POUILLET. Eléments de Physique Experimentale. 3 vols. 8vo. Hf. cf.	7.50
SERRET. Cours d'Algèbre supérieure. 8vo. Hf. cf.	3.50
Eléments d'Arithmétique. 8vo. Hf. cf.	1.50
TERQUEM. Nouvelles Annales de Mathématiques. Monthly. Per annum	4.00
Nouvelles Annales de Mathématiques, 1856-7. 2 vols. Hf. cf.	6.50
Nouvelles Annales de Mathématiques. Tones VIII. à XVII. Corresponding to the Years 1849-1859. 10 vols. Hf. cf.	
Bulletin Bibliographique, d'Histoire, et de Biographie Mathématiques. 8vo. Hf. cf.	3.00
TRESCA. Géométrie Descriptive. 8vo. Hf. mor.	3.00
VERHULST. Traité élémentaire des Fonctions Elliptiques. 8vo. Hf. cf.	8.50
VIEILLE. Cours complémentaire d'Analyse et de Mécanique. 8vo. Hf. cf.	2.50

MATHEMATICAL BOOKS.

MATHEMATICAL BOOKS RECENTLY RECEIVED.

AIRY. Mathematical Tracts. 4th ed.	\$4.50	LAGRE. Calcul des Probabilités et Théorie des Erreurs . . .	\$1.25
BIENAYME. La Probabilité des Erreurs d'après la Méthode des Moindres carrés	1.12	LUNN. Treatise on Motion. 8vo.	2.25
BOOLE. Calculus of Finite Differences	3.00	Mathematician (The). Edited by Davies, Rutherford, and Fenwick. 3 vols. 8vo.	9.00
BOURDAIS. Traité Pratique de la Résistance des Matériaux. 8vo. hf. cf.	2.75	MEISSAS. Exécution des Chemins de Fer. 12mo. hf. cf.	3.00
BRESSE. Cours de Mécanique Appliqués. 8vo. hf. cf.	3.00	NOVGENSKI ROCH. Nouvelle Théorie des Proportions et Progressions Harmoniques avec ses Applications à la Géométrie	1.00
BRIOT ET BOUQUET. Théorie des Fonctions Doublement Périodiques et en particulier des Fonctions Elliptiques. 8vo. hf. cf.	2.75	PEACOCK. Algebra	Each 2.25
BROUGHAM (LORD). Tracts, Mathematical and Physical. 8vo.	2.25	POTTER. Physical Optics. 2 parts. 8vo.	2.25
CARNOT. Reflexions sur la Méta physique du Calcul Infinitesimal	1.25	— Treatise on Hydrostatics. 8vo.	2.25
CONSOLIN. Manuel du Voilier. 8vo. paper	3.50	PRATT. Theory of the Attraction of the Earth	2.25
DIDION. Calcul des Probabilités Appliquées au tir des Projectiles	1.00	PUCKLE. Conic Sections	2.25
FRANCEUR. Traite de Géométrie	3.00	TODHUNTER. Algebra	2.25
GOODWIN. Course of Mathematics	4.25	— Plane Trigonometry	1.50
Lady and Gentleman's Diary, Poetical and Mathematical, for 189040	— Spherical Trigonometry	1.37
LAME. Leçons sur les Co-ordonnées Curvilignes et leurs diverses Applications. 8vo. hf. cf.	2.25	— Analytical Geometry, Examples of	1.25
		— Conic Sections	3.00

TERQUEM. Nouvelle Annales de Mathématiques, 1842-1859. 18 vols. Hf. cf.

Nice Copy \$50.00

SECOND-HAND MATHEMATICAL BOOKS.

AIRY. Trigonometry. 12mo.	\$0.75	HYMER. Integral Calculus. 3d ed. 8vo. hf. cf.	\$4.00
BLAND. Algebraical Problems. 8vo.	1.75	— Astronomy. 2d ed. 8vo.	2.00
— Key to do. 8vo.	1.75	LACHOIX. Elements of Algebra, translated by Prof John Farrar. 5th ed. 8vo.	1.50
— Philosophical Problems. 8vo.	1.75	Ladies' Diary. Poetical and Mathematical. 1781-1790. 16 no.	1.00
— Geometrical Problems. 8vo.	1.75	LAPLACE. Elementary Illustrations of Celestial Mechanics. 8vo.	1.25
— Elements of Hydrostatics	1.50	— Analytical Mechanics, Translated by Rev. John Topham.	1.00
DE MORGAN. The Globes, Celestial and Terrestrial	1.25	LARDNER. System of Algebraic Geometry. 8vo.	1.75
Diarian Miscellany, Mathematical and Poetical, by Hutton. 5 vols. 12mo. cf.	4.00	— Elements of Euclid. 5th Edition. 8vo.	1.50
Cambridge Problems. From 1801 to 1890. 8vo.	1.00	— Differential and Integral Calculus. 8vo.	2.00
EARNshaw. Dynamics. 8vo.	2.50	LEYBOURN. Mathematical Questions with Solutions. 4 vols. 8vo.	8.00
GRANT. Plane Astronomy. 8vo.	1.25	— Mathematical Institutions. 4to.	1.50
GREGORY AND WALTON. Application of Analysis to Solid Geometry. 8vo.	3.00	Mathematical Register; or, Diarian Repository. 4to.	2.50
GREGORY. Differential and Integral Calculus, by Walton. 8vo. 3d ed.	4.50	Mathematical Companion, The Gentleman's, for the Years 180-11-12-14-15-17-18-19-20-21-22-23. 16mo. Unbound. 11 Nos.	2.00
HERSCHEL. Calculus of Finite Differences. 8vo.	4.00	MOIGNO. Leçons de Calcul Differential. Vol. I. Hf. cf. Scarce.	9.00
HUTTON'S MATHEMATICS. Davies's Solutions of. 8vo.	2.00	MOSELEY. Hydrostatics and Hydrodynamics. 8vo. Scarce.	4.00
— Dawling's Key to. 8vo.	2.00	POISSON. Traité de Mécanique. 2 vols. 8vo. hf. bd.	5.00
HYMER. Conic Sections. 3d ed. 8vo.	2.00	PRICE. Differential Calculus. Vol. I. 1st ed.	1.50
— Analytical Geometry of Three Dimensions. 1st ed. 8vo.	1.00	PUCKLE. Conic Sections. 1st ed. 12mo.	1.00
— do. do. 2d ed. 8vo.	1.50	STEVENSON. Algebraic Equations. 8vo.	1.03
— Plane and Spherical Trigonometry. 2d ed. 8vo.	1.25	TODHUNTER. Analytical Statics. 12mo.	2.00
— do. do. 3d ed. 8vo.	1.50	WAUD. Algebraic Geometry. 8vo. Hf. cf.	1.50
		YOUNG. Algebraical Equations. 12mo.	1.00

Orders for the Importation of English, French, and German Books will be promptly executed, and at moderate prices. Early copies of all the new mathematical books published in England and France will be received as soon as published.

BOOKS FOR PUBLIC LIBRARIES IMPORTED FREE OF DUTY.

TEXT-BOOKS USED IN HARVARD COLLEGE.

THE ORATION OF ÆSCHINES AGAINST CTESIPHON. With Notes, by PROFESSOR J. T. CHAMPLIN. 12mo. cloth.	\$0.88	CONSTITUTIONAL DOCUMENTS OF ENGLAND AND AMERICA, from Magna Charta to the Federal Constitution of 1789. Compiled and Revised, with Notes, by F. BOWEN. Professor of Moral Philosophy in Harvard College. 8vo. cloth88
THE CLOUDS OF ARISTOPHANES. With Notes, by PROFESSOR C. C. FELTON. 12mo. Cloth	1.00	HODGES, R. M. PRACTICAL DISSECTIONS. By RICHARD M. HODGES, M. D., Demonstrator of Anatomy in the Medical Department of Harvard College. 16mo.	1.00
THE BIRDS OF ARISTOPHANES. With Notes, by PROFESSOR C. C. FELTON. 12mo. cloth.	1.00	HORACE. With Notes, by A. J. MACLEANE, M. A. Revised and Edited by R. H. CHASE, A. M. With Introduction to the Metre, by PROFESSOR CHARLES BECK. Revised edition. 12mo. half mor.	1.25
THE PANEGYRICUS OF ISOCRATES. With Notes, by PROFESSOR C. C. FELTON. 12mo. Cloth75	LATHAM'S ELEMENTARY ENGLISH GRAMMAR. Revised by PROFESSOR CHILDS. With an Appendix, by PRESIDENT GOODWIN, Trinity College. 16mo. Cloth75
SELECTIONS FROM THE GREEK HISTORIANS. With Notes, by PROFESSOR C. C. FELTON. 12mo. half mor.	1.50	PEIRCE, J. M. ANALYTIC GEOMETRY. 8vo.	1.50
SELECTIONS FROM MODERN GREEK WRITERS, IN PROSE AND POETRY. With Notes, by PROFESSOR C. C. FELTON. 12mo. Cloth	1.00	ROELKER, B. A GERMAN READER FOR BEGINNERS. Compiled by BERNARD ROELKER, A. M., Instructor in Harvard University. 12mo. Cloth	1.00
THE AIAS OF SOPHOKLES. With Critical and Explanatory Notes, by J. B. M. GRAY. 12mo. cloth.	1.25	SYNTAX OF THE MOODS AND TENSES OF THE GREEK VERB. By W. W. GOODWIN, Ph. D., Prof. in Harvard College. 12mo. Cloth	1.25
PRONUNCIATION AND HISTORY OF THE GREEK ALPHABET. By E. A. SOPHOCLES, Professor in Harvard College. 12mo. Cloth	1.00	VERNOV, EDWARD JOHNSTON, B. A. GUIDE TO THE ANGLO-SAXON TONGUE. 12mo. Cloth	1.25
CICERO. BRUTUS. Edited, with Notes, by PROFESSOR C. BECK. 16mo. Cloth75	WHATELY. LESSONS ON MORALS AND CHRISTIAN EVIDENCES. Edited by REV. F. D. HUNTINGTON. 16mo.75
CICERO. Immortality of the Soul; The Tusculan Disputations, Book First; The Dream of Scipio; and Extracts from the Dialogues on Old Age and Friendship. With English Notes, by THOMAS CHASE, Tutor in Harvard College. 16mo. Cloth75		
CICERO. PRO CLUENTIO. With English Notes, by AUSTIN STICKNEY, M. A., Professor of Latin in Trinity College. 12mo. Cloth75		

PUBLISHED AND FOR SALE BY

SEVER & FRANCIS,
Booksellers to the University, Cambridge.

Greenleaf's Course of Mathematics,

FOR ALL GRADES OF SCHOLARS,

IN SIX BOOKS.

FOR COMMON SCHOOLS.

1. New Primary Arithmetic.
2. Intellectual Arithmetic.
3. Common School Arithmetic.

HIGH SCHOOLS AND ACADEMIES.

4. National Arithmetic.
5. Treatise on Algebra.
6. Elements of Geometry.

The Author and Publishers of this series have spared no labor and expense to adapt the books to meet the demands of the present day, — with its improved methods of instruction.

The series, in its revised and improved form, surpasses all others, —

1. In the lucid, progressive, and strictly philosophical and scientific ARRANGEMENT of the several parts and subjects.

2. In the clearness and conciseness of the DEFINITIONS and RULES, and in the completeness of the ILLUSTRATIONS.

3. In the variety and practical character of the PROBLEMS; which, from their nature, tend to interest the pupil, exercise his ingenuity, and secure useful mental discipline.

4. In being a CONSECUTIVE SERIES, graded to the wants of PRIMARY, INTERMEDIATE, GRAMMAR and HIGH SCHOOLS, ACADEMIES, NORMAL SCHOOLS, and COMMERCIAL COLLEGES.

Greenleaf's Arithmetics are now widely used throughout the United States, by the best Educators, and the sale of them is greater than that of any other series published; — an indication that the books possess enduring merits.

The attention of Teachers of the Higher Schools is invited to the ELEMENTS OF GEOMETRY, recently published. It has received the commendation of many eminent Teachers, a few of which are subjoined.

PROF J. G. HOTT, Chancellor of Washington University, St. Louis, writes: "After a somewhat careful examination of Greenleaf's Geometry, I have decided to subject it to the test of the Recitation-Room. Of course I should not do this if I were not pretty well satisfied of its excellence as a text-book. It strikes me that there is some improvement upon Legendre in the elementary portions of the work; but the chief points of attraction to me are the copious and well-considered Problems in the Eleventh, Twelfth, and Thirteenth Books."

From A. M. GAY, A. M. Principal of the High School, Charlestown, Mass.

"From an examination, and a practical test in some of my classes, of the advanced sheets of Greenleaf's Geometry I have no hesitation in pronouncing it superior to any class-book of the kind with which I am acquainted. The basis of the work is Legendre's Elementary Geometry, which has been long and favorably known to teachers. Much valuable matter, however, has been added by the present editor, so that it cannot be regarded as a translation, or a mere compilation. The language in which the propositions are stated is remarkably clear and concise. The development of principles is sufficiently extended, and in all respects it seems a book well adapted to the purposes of instruction in Academies, High Schools, and other institutions of learning."

From D. B. HAGAR, A. M., President of the American Inst. of Instruction, etc.

"Having carefully read Greenleaf's Geometry, the work appears to me to be one of great merit. Of course no person can expect to find much that is essentially new in a geometrical treatise published at this day. But Mr. Greenleaf's, while it is based like nearly all American Geometries, upon Legendre's original work, contains valuable propositions not usually found in similar publications, and also presents uncommonly clear demonstrations of many of the standard propositions. The definitions contained in this work are generally concise, clear, and exact. The exercises in Mensuration, and in the Application of Algebra to Geometry, will be found unusually interesting and useful. In a word, I have no doubt that Mr. Greenleaf's Geometry will be found to compare favorably with the best Geometries yet published."

* * * All interested are respectfully invited to correspond freely with us, and to examine the revised, permanent, stereotyped editions. OUR DESCRIPTIVE CATALOGUE furnished, on application, by mail.

ROBERT S. DAVIS & CO.,
Publishers, Boston.

VALUABLE SCIENTIFIC WORKS

PUBLISHED BY

LITTLE, BROWN, & CO.,

110 Washington St., Boston.

PEIRCE'S ANALYTICAL MECHANICS.

In One Vol. 4to. Price \$7.50.

PHYSICAL AND CELESTIAL MECHANICS: Developed in Four Systems of Analytical Mechanics, Celestial Mechanics, Potential Physics, and Analytical Morphology. By BENJAMIN PEIRCE. Part I. Analytical Mechanics.

"I have re-examined the memoirs of the great geometers, and have striven to consolidate their latest researches and their most exalted forms of thought into a consistent and uniform treatise. If I have, hereby, succeeded in opening to the students of my country a readier access to these choice jewels of intellect; if their brilliancy is not impaired in this attempt to reset them; if, in their own constellation, they illustrate each other, and concentrate a stronger light upon the names of their discoverers; and, still more, if any gem which I may have presumed to add, is not wholly lustreless in the collection, I shall feel that my work has not been in vain." — *Extract from the Preface.*

GAUSS'S THEORIA MOTUS.

In One Vol. 4to. Price \$5.

THEORY OF THE MOTIONS OF THE HEAVENLY BODIES MOVING ABOUT THE SUN IN CONIC SECTIONS. A translation of Gauss's Theoria Motus, with an Appendix. By CHARLES HENRY DAVIS.

COOKE'S CHEMICAL PHYSICS.

In One Vol. 8vo. Price \$3.

ELEMENTS OF CHEMICAL PHYSICS. By JOSIAH P. COOKE, Jr., Erving Professor of Chemistry and Mineralogy, in Harvard College. Illustrated by 400 Woodcuts. With a copious Index.

The Mathematical Monthly.

In commencing our third volume, we beg to assure the friends of this Journal, that all legitimate efforts will be made to sustain it, together with the Prizes, which have proved such an excellent feature of its plan. It will be seen (see 2d page of cover) that the amount of the Prizes has been increased. But all efforts on our part must prove unavailing if the subscription list, with other sources of income, will not cover the expenses which we are incurring to make the Monthly really valuable and attractive to all interested in it. To show our friends at home what is thought of the Monthly abroad, we venture to make short extracts from letters lately received.

"The work appears to me so excellent, that I have at once ordered the series for the Library of the Observatory. G. B. AIRY, Ast. Royal."

"Royal Obs., Greenwich, July 5, 1860.

"I have looked through it with great interest, and especially because it shows to what an extent interest in mathematical subjects is felt through the United States. I do not believe that it would be possible in England to get up a Mathematical periodical brought out in so handsome a form and supported by so large a number of subscribers. GEORGE SALMON."

"Trinity College, Dublin, July 12, 1860.

"DEAR SIR, — I thank you sincerely for the copy you so kindly sent me of the first volume of the Mathematical Monthly. We have not, that I am aware of, any work of a similar character in England. I feel confident it will serve a valuable purpose in stimulating and giving a healthy tone to the study of Mathematics in the higher class of schools, and judging from this volume, it will be a work of as great interest and give as much matter for thought to the tutor as to the student. Original investigations on the mere elementary branches of science are perhaps not often to be expected; those portions have now been so long and so frequently considered, and by so many different minds, that something new — a really original idea — appears only at very distant intervals. Still many interesting questions may, and, as your first volume shows, do arise even in the earliest branches of Mathematical science; whether discussions on the principles and on the best methods of placing them before the pupil, or examples and problems which in their solutions show a large amount of ingenuity and mathematical skill. The Mathematical Monthly claims a special interest from the very fact of its character rendering it useful to the many without detracting from its scientific merit."

"I remain, your obedient servant."

"St. John's College, Cambridge, July 26, 1860.

HUGH GODFRAY."

We feel confident that a little effort on the part of our friends would double our subscription list. See terms to clubs and new subscribers taking Vols. I., II., III. We could then engrave some of the fine portraits of LAGRANGE, MONTE, CAUCHY, BIOR, &c., lately received; and, indeed, incur any expense to increase the value of the Monthly to all. Will our friends make up their clubs at once, and notify us, so that we can decide upon the size of our edition?

IMPORTANT EDUCATIONAL WORKS

RECENTLY PUBLISHED BY

GOULD & LINCOLN, BOSTON.

Lectures on Logic. By Sir William Hamilton, Bart., Professor of Logic and Metaphysics in the University of Edinburgh. With Notes from original materials, and an Appendix containing the latest development of his new Logical Theory. Edited by H. Longueville Mansel, B. D., Oxford, and John Veitch, A. M., Edinburgh. Royal 8vo. Cloth, \$3.00.

This volume completes the Metaphysical and Logical series of Sir William Hamilton, and will enable American scholars to possess themselves of the invaluable works of this eminent author, in two elegant octavo volumes, at six dollars, — half the price of the English edition.

Lectures on Metaphysics. By Sir William Hamilton, Bart. Edited by Prof. H. L. Mansel, B. D., Oxford, and John Veitch, M. A., Edinburgh. Royal 8vo. Cloth, \$3.00.

Hamilton's Lectures on Metaphysics have been received with eminent favor and already adopted as a text-book in Harvard, Yale, Princeton, and other distinguished Colleges.

Prolegomena Logica. An Inquiry into the Psychological Character of Logical Processes. By H. Longueville Mansel, B. D., Professor of Moral and Metaphysical Philosophy, Oxford; Editor of Sir William Hamilton's Lectures; Author of "Limits of Religious Thought," etc. 12mo. Cloth, \$1.00.

This work is not an Introduction to Logic, but "an inquiry into the constitution and laws of the thinking faculty, such as they are assumed by the logician as the basis of his deductions." It is an important contribution to Mental Science, by one who stands in the front rank of living authors in this department of learning.

Limits of Religious Thought Examined. By H. Longueville Mansel, B. D., Professor of Moral and Metaphysical Philosophy, Oxford; Editor of Sir William Hamilton's Lectures, etc. 12mo. Cloth, \$1.00.

This able work of Prof. Mansel has passed through several editions, and is attracting marked attention among scholars and theologians, many of whom regard it as a defence of revealed religion unsurpassed since the appearance of Butler's Analogy.

Mental Philosophy; Including the Intellect, Sensibilities, and Will. By Joseph Haven, Professor of Intellectual and Moral Philosophy, Amherst College. Royal 12mo. Cloth, \$1.50.

Prof. Park. of Andover, says, "It is distinguished for its clearness of style, perspicuity of method, candor of spirit, acumen, and comprehensiveness of thought. I have been heartily interested in it."

"Earnestness, candor, and accuracy and elegance of diction characterize every page, and render the work as interesting to the general reader as it is valuable to the scholar." — D. H. COCHRAN. *Principal and Prof. of Moral and Intellectual Philosophy, State Normal School, Albany.*

This work has been extensively adopted as a text-book in Colleges and High Schools throughout the country.

Moral Philosophy; Including Theoretical and Practical Ethics. By Joseph Haven, D. D., late Professor of Moral and Intellectual Philosophy in Amherst College; Author of "Mental Philosophy." Royal 12mo. Cloth, \$1.25.

It is scientific in method and thorough in discussion, and its views on unsettled questions in morals are discriminating and sound. It treats largely of Political ethics, — a department of morals of great importance to American youth, but generally overlooked in text-books. In the history of ethical opinions, it is unusually rich and elaborate.

Elements of Moral Science. By Francis Wayland, D. D., late President of Brown University. 12mo. Cloth, \$1.25.

Elements of Political Economy. By Francis Wayland, D. D., 12mo. Cloth, \$1.25.

All the above works by Dr. Wayland are used as text-books in many of the colleges and higher schools throughout the Union, and are highly approved.

The Elements of Geology; Adapted to Schools and Colleges. With numerous Illustrations. By J. R. Loomis, President of Lewisburg University, Pa. 12mo. Cloth, 75 cents.

"It is surpassed by no work before the American public." — M. B. ANDERSON, LL. D., *President of Rochester University.*

Principles of Zoology; Touching the Structure, Development, Distribution, and Natural Arrangement of the RACES OF ANIMALS, Living and Extinct; with numerous Illustrations. For the use of Schools and Colleges. Part I. COMPARATIVE PHYSIOLOGY. By Louis Agassiz and Augustus A. Gould. Revised edition. \$1.00.

Thesaurus of English Words and Phrases; so classified and arranged as to facilitate the expression of ideas and assist in literary composition. By Peter Mark Roget, late Secretary of the Royal Society, London, etc. Revised and edited, with a List of Foreign Words defined in English, and other additions, by Barnas Sears, D. D., President of Brown University. A New American Edition, with Additions and Improvements. 12mo. Cloth, \$1.50.

The Earth and Man; Lectures on Comparative Physical Geography, in its relation to the History of Mankind. By Arnold Guyot. With Illustrations. 12mo. Cloth, \$1.25.

Prof. Louis Agassiz, of Harvard University, says, "It will not only render the study of Geography more attractive, but actually show it in its true light."

Essays in Biography and Criticism. By Peter Bayne, M. A., Author of "The Christian Life, Social and Individual." Arranged in two Series or Parts. 12mo. Cloth, \$1.25 each.

Guyot's Mural Maps. A series of Elegant Colored Maps for the Recitation Room, exhibiting the Physical Phenomena of the Globe. By Prof. Arnold Guyot, viz.: Map of the World, mounted, \$10.00. Map of North America, mounted, \$9.00. Map of South America, mounted, \$9.00. Map of Geographical Elements, mounted, \$9.00.

Chambers's Cyclopædia of English Literature. A Selection of the choicest productions of English Authors, from the earliest to the present time. Connected by a Critical and Biographical History. Two imperial octavo volumes, 700 pages each, double column; with 300 elegant Illustrations. Embossed Cloth, \$5.00.

The Life of John Milton, Narrated in Connection with the Political, Ecclesiastical, and Literary History of his Time. By David Masson, M. A., Professor of English Literature, University College, London. Vol. I, 1608 to 1639. With Portraits, &c. Royal 8vo. Cloth, \$2.75.

An elaborate, erudite, and magnificent work.

Annual of Scientific Discovery for 1860; or, Year-Book of Facts in Science and Art, exhibiting the most important Discoveries and Improvements in Mechanics, Useful Arts, Natural Philosophy, Chemistry, Astronomy, Meteorology, Zoology, Botany, Mineralogy, Geology, Geography, Antiquities, &c. Edited by David A. Wells, A. M. 12mo. Cloth, \$1.25.

The Puritans; or the Court, Church, and Parliament of England, during the reigns of Edward VI. and Elizabeth. By Samuel Hopkins, Author of "Lessons at the Cross," etc. In 3 vols. Vols. I and II. now ready. 8vo. Cloth, \$2.50 per vol.

This is a masterly work, by one of the most distinguished of living authors.

Cyclopædia of Anecdotes of Literature and the Fine Arts. Containing a copious and choice Selection of Anecdotes of the various forms of Literature, of the Arts, of Architecture, Engravings, Music, Poetry, Painting and Sculpture, and of the most celebrated Literary Characters and Artists, &c. By Kazlitt Arvine, A. M., Author of "Cyclopædia of Moral and Religious Anecdotes." With numerous Illustrations. 725 pp. 8vo. Cloth, \$3.00.

The Greyson Letters. Selections from the Correspondence of R. E. H. Greyson, Esq. Edited by Henry Rogers, Author of "The Eclipse of Faith." 12mo. Cloth, \$1.25.

"A book not for one hour, but for all hours; not for one mood, but for every mood; to think over, to dream over, to laugh over." — *Boston Journal.*
"The Letters are intellectual gems, radiant with beauty, happily intermingling the grave and the gay." — *Christian Observer.*

Knowledge is Power. A View of the Productive Forces of Modern Society, and the Results of Labor, Capital, and Skill. By Charles Knight. With numerous Illustrations. Revised, with Additions, by David A. Wells, Editor of the "Annual of Scientific Discovery." 12mo. Cloth, \$1.25.

"This is emphatically a book for the people. It contains an immense amount of important information, which everybody ought to be in possession of; and the volume should be placed in every family and in every School and Public Library in the land."

Hugh Miller's Works. Seven volumes, uniform style, in an elegant box. Embossed Cloth, \$8.25.



f
-
-
y

i,
-
L.

s.
e
es
e,
e
itt
us
h,

of
he

for
al.
er-

of
ill.
ith
ific

ense
ses-
ool

an

P. 1 of 86